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Using Some Reducing Techniques to Simplify Reliability Network

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Abstract. In our research we will present three reduction strategies to calculating the reliability of complex network. In the first method we systematically update every parallel path via an equal single path, and ultimately lessen the given system to one together with most effective series detail, which is called reduction to series elements. The second is the decomposition method which is an application of the law of total probability which is includes choosing a key component and then calculating the reliability of the network when the key failed once and the second though the key succeeded. These two probabilities are then blended to obtain the reliability of the system. And the third method is the delta-star transformation. All of these methods convert a complex network into simple network.
Keywords: reliability, network, decomposition, reduction to series, delta-star.

INTRODUCTION

Reliability theory describes the ability of a network to complete the task for which it is responsible at a specific time. It is one of the pillars of engineering. Network reliability evaluation gets superb attention for the planning, effectiveness, and protection of many real international networks, such as computers, communications, electrical circuits, aircraft, linear accelerators, and power networks [1, 2]. The devices of a network are subject to random failures, as many groups and institutions become dependent upon networked computing packages [3]. The failure of any unit of a network may additionally at once have an effect on the operation of a network, for this reason the probability of every unit of a network is a completely important whilst considering the reliability of a network [3,4]. Hence the reliability consideration is an essential aspect in networked computing. There are number of ways to calculate the reliability of complex network. Such are, for examples, decomposition method, reduction to series elements technique and delta-star transformation which are depend on the digraph [4,6]. The above-mentioned methods reduce complex networks and convert them into simple networks for easy reliability calculation, as will be evidenced by our discussion of those methods.

DEFINITIONS AND NOTES

Probability

A probability is the chance of an event occurring [1,4]. The probability of an event A denoted by $Pr(A)$. For any event A we have $0 \leq Pr(A) \leq 1$.

Conditional Probability

Let A, B be two events. Then the conditional probability $Pr(A|B)$ is defined as the probability of an event A occurring knowing that another event B has already occurred [4,6]. That is $Pr(A|B) = Pr(A \cap B)/Pr(B)$.

Digraph

Let $N(G)$ is the set of nodes such that they are related by a set of directed edges $E(G)$[3,7]. Then $G = (N(G), E(G))$ is called a digraph.

Note: $E(G)$ is representing the set of units in network that can fail with known probabilities.

Minimal path

A path is a sequence of edges of a digraph. ***A minimal path*** is a path from which no edge can be removed without disconnecting the link it creates between the begin node and the end node [2,6].

Note: A path is a set of edges which, while working, connect the begin node with the end node through operating edges, thereby guaranteeing that the network is an operating state.

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Reliability Networks

A network is a family of subnetworks, assemblies and additives organized in a specific design a good way to attain preferred features with perfect overall performance and reliability [6- 8]. The sorts of components, their qualities and the way in which they're organized inside the machine have a direct impact on the gadget's reliability. so, further to the reliability of the components, the relationship between those additives is likewise taken into consideration and decisions as to the choice of components may be made to enhance or optimize the overall network reliability, maintainability and availability [4, 7].

To evaluate the reliability of any network with many components can be by calculating the reliability of each component, and combining these reliabilities. The manner in which they're combined depends at the manner in which the components are connected, series or parallel. There are two types of networks, the first is simple network and the second is complex network to calculate the reliability of any complex network we can convert it into simple network by many methods, in our study we convert the complex network in Figure 1 into simple network [9, 10].

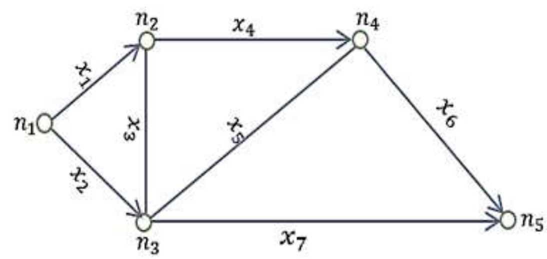

FIGURE 1. Complex network

Note that the reliability of a series and parallel networks whose have m components are respectively:

$$R_{series} = \prod_{i=1}^{m} R_i \quad (1)$$

$$R_{parallel} = 1 - \prod_{i=1}^{m} (1 - R_i) \quad (2)$$

REDUCTION TO SERIES ELEMENTS

In this technique we systematically update every parallel path via an equal single path, and ultimately lessen the given network to one together with most effective series detail. We can provide an explanation for the fixing of this approach through the following steps [10-12].

Minimal paths of our complex network are $P_1 = \{x_2 x_7\}, P_2 = \{x_1 x_3 x_7\}, P_3 = \{x_1 x_4 x_6\}, P_4 = \{x_2 x_5 x_6\}$, $P_5 = \{x_2 x_3 x_4 x_6\}$ $P_6 = \{x_1 x_3 x_5 x_6\}, P_7 = \{x_1 x_4 x_5 x_7\}$ and $P_8 = \{x_2 x_3 x_4 x_5 x_7\}$.

Step 1: The network has been changed to another network as proven in Figure 2.

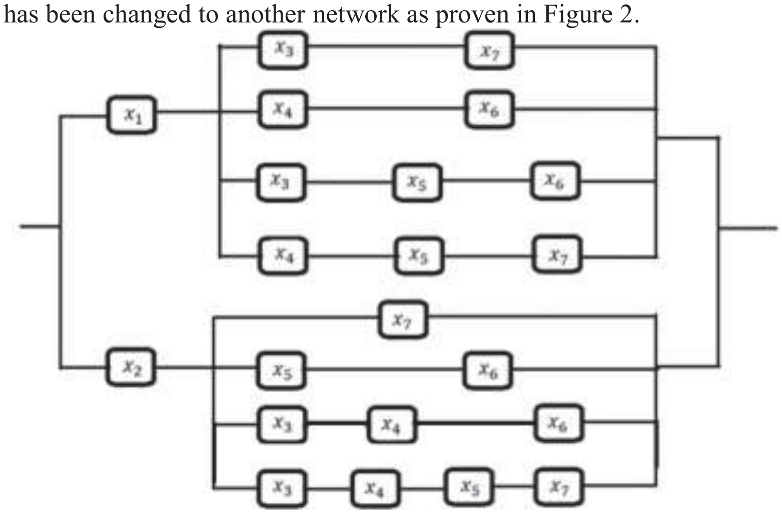

FIGURE 2. Series- parallel network

Step 2: Partition the network for two series- parallel subsystems (1), (2) as in Figure 3 and find the reliability of them to get:

$$R_{subsystem\,1} = 1 - (1 - R_3R_7)(1 - R_4R_6)(1 - R_3R_5R_6)(1 - R_4R_5R_7) \quad (3)$$

$$R_{subsystem\,2} = 1 - (1 - R_7)(1 - R_5R_6)(1 - R_3R_4R_6)(1 - R_3R_4R_5R_7) \quad (4)$$

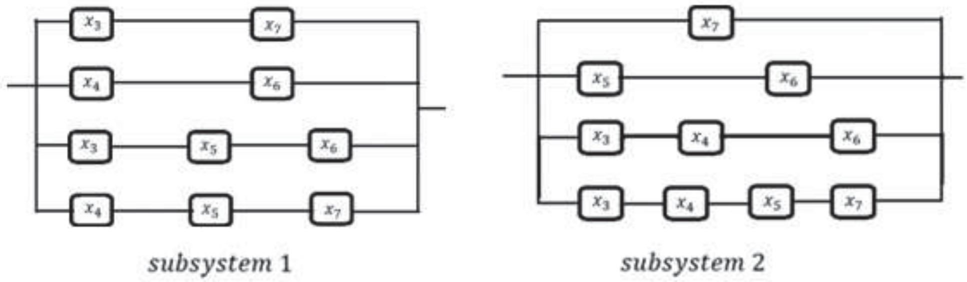

FIGURE 3. Subsystems (1) and (2)

Step 3: Now the network has been decreased to a device contains series- parallel elements which is equivalents to our network as in Figure 4.

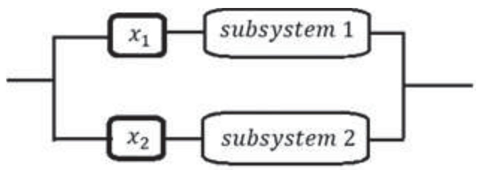

FIGURE 4. Subsystem (3)

Then the reliability of the subsystem (3) which is equivalent to complex network in figure (1) as follows

$$R = 1 - (1 - R_1R_{subsystem\,1})(1 - R_2R_{subsystem\,2}) \quad (5)$$

$$R = 1 - \left[1 - R_1\left(1 - (1 - R_3R_7)(1 - R_4R_6)(1 - R_3R_5R_6)(1 - R_4R_5R_7)\right)\right.$$
$$\left.\left(1 - R_2\left(1 - (1 - R_7)(1 - R_5R_6)(1 - R_3R_4R_6)(1 - R_3R_4R_5R_7)\right)\right)\right] \quad (6)$$

THE DECOMPOSITION METHOD

The decomposition method is an application of the law of total probability. It includes choosing one component as "key" and then calculating the reliability of the network twice: the first though the important thing factor failed $(R = 0)$ and the second though the important thing element succeeded $(R = 1)$. These two probabilities are then blended to obtain the reliability of the network, considering the fact that at any given time the important thing could be failed or running. Using probability idea, the equation is [11,12]

$$R_S(t) = Pr(S \cap C) + Pr(S \cap \bar{C}) \quad (7)$$

where C is key component and $\bar{C}$ the complement of C.

By using the definition of conditional probability, equation (7) becomes:

$$R_S(t) = Pr(S|C)Pr(C) + Pr(S|\bar{C})Pr(\bar{C}) \quad (8)$$

Now applying equation (4) to calculate the reliability of our network

$$R_S(t) = Pr(S|S_3)Pr(S_3) + Pr(S|\bar{S_3})Pr(\bar{S_3}) \quad (9)$$

where S_3 is the "key" (i.e. $x_3 = 1$ if S_3 is working, and $x_3 = 0$ if S_3 is failed).

If S_3 works, then the network $(S|S_3)$ becomes as in Figure 5 which have an equivalent two diagrams.

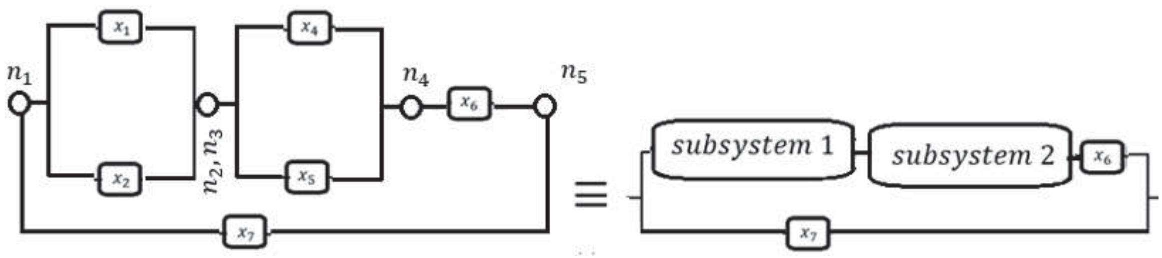

FIGURE 5. Network $(S|S_3)$

Hence, the conditional probability $Pr(S|S_3)$ is:

$$Pr(S|S_3) = 1 - \left(1 - R_{subsystem\ 1}\right)\left(1 - R_{subsystem\ 2}\right)R_6(1 - R_7) \tag{10}$$

$$Pr(S|S_3) = 1 - (1 - R_7)\left[1 - \left(1 - (1 - R_1)(1 - R_2)\right)\right]\left[1 - \left(1 - (1 - R_4)(1 - R_5)\right)\right]R_6 \tag{11}$$

If S_3 fails, then the network $(S|\bar{S_3})$ becomes as in Figure 6.

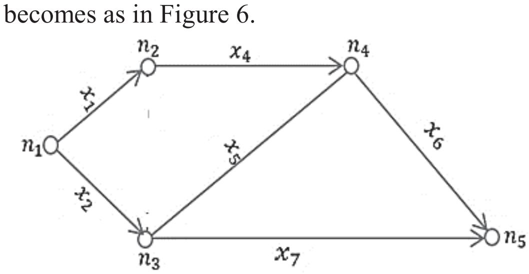

FIGURE 6. Network $(S|\bar{S_3})$

So, the probability of $(S|\bar{S_3})$ is

$$Pr(S|\bar{S_3}) = 1 - (1 - R_1R_4R_6)(1 - R_2R_7)(1 - R_2R_5R_6)(1 - R_1R_4R_5R_7) \tag{12}$$

Therefore, the reliability of our network is giving by:

$$R_S = R_3\left[1 - (1 - R_7)\left[1 - \left(1 - (1 - R_1)(1 - R_2)\right)\right]\left[1 - \left(1 - (1 - R_4)(1 - R_5)\right)\right]R_6\right]$$
$$+ (1 - R_3)\left[1 - (1 - R_1R_4R_6)(1 - R_2R_7)(1 - R_2R_5R_6)(1 - R_1R_4R_5R_7)\right] \tag{13}$$

Note: Reliability for the network in Figure 1 can be calculated by choosing more than one key's component (for more details, see [13]).

DELTA-STAR METHOD

For analyze any complex network consisting triangle we can using the delta-star transformation without difficulty transforms the arrangement into series and parallel combinations. By means of acquiring the equivalent parts of two diagrams in Figure 7 we can find the delta-star equations [7, 14].

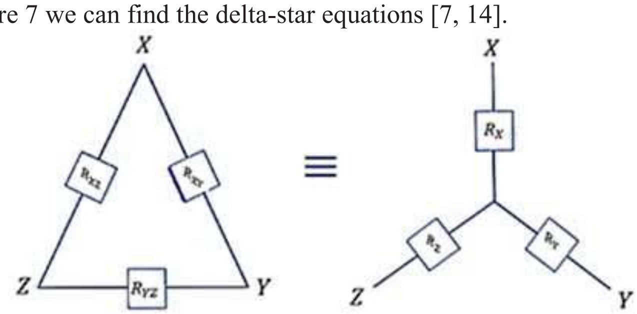

FIGURE 7. Delta-star diagrams.

Consider the sides of the triangle with reliabilities R_{XY}, R_{XZ} and R_{YZ} connected to shape the delta arrangement shown in left diagram above [7, 15]. This arrangement yields the star equivalent with reliabilities R_X, R_Y and R_Z as in right diagram.

If we applying the equations (1) and (2) to two parts of two diagrams in Figure 7, then we get three relations:

$$R_X R_Y = 1 - (1 - R_{XY})(1 - R_{XZ}R_{YZ}) \quad (14)$$

$$R_X R_Z = 1 - (1 - R_{XZ})(1 - R_{XY}R_{YZ}) \quad (15)$$

$$R_Y R_Z = 1 - (1 - R_{YZ})(1 - R_{XY}R_{XZ}) \quad (16)$$

By solving the three relations above we get the following delta-star relationships

$$R_X = \sqrt{\frac{\alpha\beta}{\gamma}} \quad , \quad R_Y = \sqrt{\frac{\alpha\gamma}{\beta}} \quad \text{and} \quad R_Z = \sqrt{\frac{\beta\gamma}{\alpha}} \quad (17)$$

Where $\alpha = 1 - (1 - R_{XY})(1 - R_{XZ}R_{YZ})$,$\beta = 1 - (1 - R_{XZ})(1 - R_{XY}R_{YZ})$ and $\gamma = 1 - (1 - R_{YZ})(1 - R_{XY}R_{XZ})$

Applying the delta-star relationships to the triangle $n_1n_2n_3$ in Figure 1 transfers the complex network to a simple [16, 17]. So, the complex network in Figure 1 becomes as in the Figure 8.

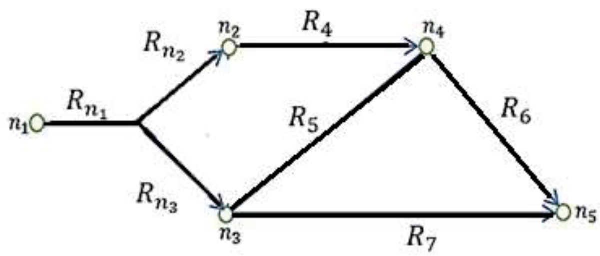

FIGURE 8. Modified network.

Now applying the relationships in equation (17) to find R_{n_1}, R_{n_2} and R_{n_3}

$$R_{n_1} = \sqrt{\frac{[1-(1-R_1)(1-R_2R_3)][1-(1-R_2)(1-R_1R_3)]}{1-(1-R_3)(1-R_1R_2)}}$$

$$R_{n_2} = \sqrt{\frac{[1-(1-R_1)(1-R_2R_3)][1-(1-R_3)(1-R_1R_2)]}{1-(1-R_2)(1-R_1R_3)}}$$

$$R_{n_3} = \sqrt{\frac{[1-(1-R_2)(1-R_1R_3)][1-(1-R_3)(1-R_1R_2)]}{1-(1-R_1)(1-R_2R_3)}}$$

And the reliability R_S of our network is given by

$$R_S = 1 - (1 - R_{n_1}R_{n_3}R_7)(1 - R_{n_1}R_{n_2}R_4R_6)(1 - R_{n_1}R_{n_3}R_5R_6)(1 - R_{n_1}R_{n_2}R_4R_5R_7) \quad (18)$$

Or $$R_S = 1 - (1 - R_{n_3}R_7)(1 - R_{n_2}R_4R_6)(1 - R_{n_3}R_5R_6)(1 - R_{n_2}R_4R_5R_7)R_{n_1} \quad (19)$$

There is a possibility to reduce the complex network in Figure 1 to a series network consisting of two components, the first is R_{n_1} and the second is produced by reducing the remaining network, and this is evident through the final equation resulting from the solution of the network [6, 18, 19].

COMPUTATION THE RELIABILITY

In this section we will do some computations in order to make a comparison between these methods and discover which is better by substituting random values in the final equation for each method, as well as compensating for similar values for all units for the same purpose, as follows

Case 1

we can substitution some random values for reliability to check which techniques closed to optimal value of reliability network

$(\text{i.e.}, \text{let } R_1 = 0.9\ , R_2 = 0.85, R_3 = 0.7, R_4 = 0.75, R_5 = 0.8\ , R_6 = 0.95, \text{and } R_7 = 0.8)$

Case 2

If all units have the same reliability values such as 0.8, $(\text{i.e.}, R_i = 0.8 \quad \forall\, i = 1,2,\cdots,7)$

TABLE 1. Summary table for comparison between all techniques with two cases

Methods	Case 1	Case 2
Reduction to series element	0.980	0.951
Decomposition	**0.992**	**0.987**
Delta- star	0.989	0.986

DISCUSS THE RESULTS

From table (1), we can see that the highest value of reliability of complex network Figure 1 is **0.992** and this was obtained by substituting the different values (**case 1)** into eq. (13), also, the highest value of reliability of complex network Figure 1 is **0.987** and this was obtained by substituting the similar values (**case 2)** into eq. (13).

CONCLUSION

The decomposition technique is an opportunity technique for the network's reliability analysis with an ample ability of application. It's far based on the management of conditional probability. This method includes choosing a key component, this choosing may be an accurate choice because if one chooses the key randomly, the network may be turn into a more complex network. As for in the (reduction to series elements method), we reduced the parallel paths to make the given network easier and we noticed that the reliability was calculated simply. On the other hand, in the third method (delta-star), once the complex network has been transformed into its parallel and series form, the network minimization approach is applied to calculate its reliability. We saw the ease of dealing with complex networks by using the three methods. We also noticed that when using the first and second methods, the complex network turned into a series - parallel system, while in delta-star method the network turned into a series system. Also, the most important think which can be shown that the decomposition method gives the highest value that can be reached by network reliability among the remaining methods reviewed in this paper.

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