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On new concepts of neutrosophic crisp open sets

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Abstract

Our work announces novel concepts called neutrosophic crisp i-open sets, neutrosophic crisp inter-open sets and neutrosophic crisp ii-open sets by generalizing neutrosophic crisp open set in neutrosophic crisp topological space. Numerous properties and characterizations of these concepts are studied, and the relationships of these concepts with various other concepts of neutrosophic crisp open sets are illustrated.

Subject Classification: *(2010) 03E72, 03F55, 54A40, 62C86, 68U99.*

Keywords: *NCi-OS, NCint-OS and NCii-OS.*

1. Introduction

The thought of neutrosophic crisp topological space (fleetingly NCTS) is established by Salama et al. [1]. The intellect of neutrosophic crisp semi-α-closed sets is tendered by Al-Hamido et al. [2]. The impression

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of innovative kinds of weakly neutrosophic crisp continuity is explored Imran et al. [3].

The sense of regular semiclosed sets in neutrosophic crisp topological spaces is awarded by Vadivel et al. [4]. The topic of neutrosophic α^m -continuity is displayed by Dhavaseelan et al. [5]. Some recent articles have new ideas and vital concepts in neutrosophic and topology; see Muneshwar et al. [6], Abbas et al. [7], Al-Obaidi et al. [8] and Imran et al. [9].This article intends to establish fresh conceptions by taking a broad view of neutrosophic crisp open sets and called them by neutrosophic crisp i-open sets, neutrosophic crisp inter-op neutrosophic crisp ii-open sets. Many essential properties and characterizations of these concepts are disused, and the interactions of these concepts are clarified.

2. Preliminaries

Anywhere in this article, $\mathcal{Q},\zeta$ (or frugally $\mathcal{Q}$) always mean NCTS. Let $\mathcal{F}$ be a neutrosophic crisp set (NC-set for short) in $\mathcal{Q}$. Then we have the following:

i. $\mathcal{F}^c = \mathcal{Q} - \mathcal{F}$ is referred to the neutrosophic crisp complement of $\mathcal{F}$.

ii. $NC\text{-}\mathrm{int}(\mathcal{F})$ is signified the neutrosophic crisp interior of $\mathcal{F}$.

iii. $NC\text{-}\mathrm{cl}(\mathcal{F})$ is implied the neutrosophic crisp closure of $\mathcal{F}$.

Definitions 2.1 [1] : Let non-empty set $\mathcal{Q}$ be a fixed sample space. A NC-set $\mathcal{F}$ is a thing taking the form $\mathcal{F} = \langle \mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3 \rangle$ where $\mathcal{F}_1, \mathcal{F}_2$ and $\mathcal{F}_3$ are mutually disjoint subsets of $\mathcal{Q}$.

Definition 2.2 [10] : Let $\mathcal{F}$ be a NC-subset of a NCTS $\mathcal{Q}$, then we have the following :

i. NCS-OS is signified of a neutrosophic crisp semi-open set if $\mathcal{F} \subseteq NC\text{-}\mathrm{cl}(NC\text{-}\mathrm{int}(\mathcal{F}))$.

ii. NCS-CS is represented of a neutrosophic crisp semi-closed set (The complement of a NCS-OS).

iii. $NCSO(\mathcal{Q})$ (resp. $NCSC(\mathcal{Q})$) is referred to the class of all NCS-OSs (resp. NCS-CSs) in $\mathcal{Q}$.

iv. NCα-OS is implied of a neutrosophic crisp α-open set if $\mathcal{F} \subseteq NC\text{-}\mathrm{int}(NC\text{-}\mathrm{cl}(NC\text{-}\mathrm{int}(\mathcal{F})))$.

v. NCα-CS is meant of a neutrosophic crisp α-closed set (the complement of a NCα-OS).

vi. $NC\alpha O(\mathcal{Q})$ (resp. $NC\alpha C(\mathcal{Q})$) is designated of the class of each NCα-OSs (sequentially NCα-CSs) in $\mathcal{Q}$.

Remark 2.3 [11] : The subsequent declarations hold in any NCTS $\mathcal{Q}$, but the opposite is a fallacy for each of them :

i. Each NC-OS represents a NCS-OS and NCα-OS.

ii. Each NCα-OS represents a NCS-OS.

Theorem 2.4 [10] : Leta NC-set $\mathcal{F}$ be asubset of a NCTS $\mathcal{Q}$, then $\mathcal{Q}$ iff for some NC-OS $\mathcal{S}$ where $\mathcal{S} \subseteq \mathcal{F} \subseteq NC\text{-}\mathrm{int}(NC\text{-}\mathrm{cl}(\mathcal{S}))$.

3. Neutrosophic crisp i-open sets

Definition 3.1 : A NC-subset $\mathcal{F}$ of a NCTS $\mathcal{Q}$ is indicated as a neutrosophic crisp i-open set (in brief NCi-OS) if there exist NC-OS $\mathcal{G} \in \zeta$ such that $\mathcal{G} \neq \varphi_N, \mathcal{Q}_N$, $\mathcal{F} \subseteq NC\text{-}\mathrm{cl}(\mathcal{F} \cap \mathcal{G})$. The set complement of a NCi-OS is termed a neutrosophic crisp i-closed set (in a few NCi-CS) in $\mathcal{Q}$. The collection of all NCi-OSs (resp. NCi-CSs) of $\mathcal{Q}$ is signified by $NCiO(\mathcal{Q})$ (resp. $NCiC(\mathcal{Q})$).

Example 3.2 : Let $\mathcal{Q} = \{t_1, t_2, t_3\}$. Then

$$\zeta = \{\varphi_N, \langle\{t_1\}, \varphi, \varphi\rangle, \langle\{t_2\}, \varphi, \varphi\rangle, \langle\{t_1, t_2\}, \varphi, \varphi\rangle, \mathcal{Q}_N\}$$

is a NCTS. The collection of all NCi-OSs of $\mathcal{Q}$ is :

$$NCiO(\mathcal{Q}) = \{\varphi_N, \langle\{t_1\}, \varphi, \varphi\rangle, \langle\{t_2\}, \varphi, \varphi\rangle, \langle\{t_1, t_2\}, \varphi, \varphi\rangle, \langle\{t_1, t_3\}, \varphi, \varphi\rangle, \langle\{t_2, t_3\}, \varphi, \varphi\rangle, \mathcal{Q}_N\}.$$

Proposition 3.3 : Each NC-OS is a NCi-OS.

Proof **:** Let $\mathcal{G} \in \zeta$. It is evident that $\mathcal{G} \subseteq NC\text{-}\mathrm{cl}(\mathcal{G} \cap \mathcal{G}) \Rightarrow \mathcal{G} \subseteq NC\text{-}\mathrm{cl}(\mathcal{G})$ where $NC\text{-}\mathrm{cl}(\mathcal{G})$ denotes the neutrosophic crisp closure of $\mathcal{G}$. Hence $\mathcal{G}$ is a NCi-OS.

The reverse of the above proposition is not valid as common as the next instance below.

Example 3.4 : Let $\mathcal{Q} = \{\mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3\}$. Then $\zeta = \{\varphi_N, \langle\{\mathfrak{y}_1\}, \varphi, \varphi\rangle, \langle\{\mathfrak{y}_1, \mathfrak{y}_3\}, \varphi, \varphi\rangle, \mathcal{Q}_N\}$ is a NCTS. The family of all NCi-OSs of $\mathcal{Q}$ is :

$$NCiO(\mathcal{Q}) = \{\langle\{\mathfrak{y}_1\},\varphi,\varphi\rangle,\varphi_N,\langle\{\mathfrak{y}_3\},\varphi,\varphi\rangle,\mathcal{Q}_N,\langle\{\mathfrak{y}_1,\mathfrak{y}_2\},\varphi,\varphi\rangle,\langle\{\mathfrak{y}_1,\mathfrak{y}_3\},\varphi,\varphi\rangle,\langle\{\mathfrak{y}_2,\mathfrak{y}_3\},\varphi,\varphi\rangle\}.$$

Therefore; $\langle\{\mathfrak{y}_1,\mathfrak{y}_2\},\varphi,\varphi\rangle$ is NCi-OS but this set does not form NC-OS.

Proposition 3.5 : Each NC-CS is a NCi-CS.

Proof **:** Clear. □

The reverse of proposition 3.5 is not valid in common, as the next instance show.

Example 3.6 : In example 3.4, we see that $\langle\{\varepsilon_3\},\varphi,\varphi\rangle$ is NCi-CS but is not NC-CS.

Proposition 3.7 : Each NCS-OS is a NCi-OS.

Proof **:** Assume $\mathcal{F}$ be a NCS-OS in $\mathcal{Q}$. By definition of NCS-OS there exists NC-OS $\mathcal{V}$ such that

$$\mathcal{V} \subseteq \mathcal{F} \subseteq NC - \mathrm{cl}(\mathcal{V}) \tag{1}$$

Since $\mathcal{V} \subseteq \mathcal{F}$, so we note that

$$\mathcal{F} \cap \mathcal{V} = \mathcal{V} \tag{2}$$

By (1) and (2), we obtain that $\mathcal{F} \subseteq NC\text{-}\mathrm{cl}(\mathcal{F} \cap \mathcal{V})$. Hence $\mathcal{F}$ is NCi-OS. □

The reverse of proposition 3.7 is not valid in common, as the next instance show.

Example 3.8 : Let $\mathcal{Q} = \{\varepsilon_1,\varepsilon_2,\varepsilon_3\}$. Then $\zeta = \{\varphi_N,\langle\{\varepsilon_1,\varepsilon_2\},\varphi,\varphi\rangle,\mathcal{Q}_N\}$ is a NCTS. The collection of all NCi-OSs of $\mathcal{Q}$ is :

$$NCiO(\mathcal{Q}) = \{\langle\{\varepsilon_1\},\varphi,\varphi\rangle,\langle\{\varepsilon_2\},\varphi,\varphi\rangle,\varphi_N,\langle\{\varepsilon_1,\varepsilon_2\},\varphi,\varphi\rangle,\mathcal{Q}_N,\langle\{\varepsilon_1,\varepsilon_3\},\varphi,\varphi\rangle,\langle\{\varepsilon_2,\varepsilon_3\},\varphi,\varphi\rangle\}.$$

We see that $\langle\{\varepsilon_1,\varepsilon_3\},\varphi,\varphi\rangle$ is NCi-OS but is not NCS-OS.

Proposition 3.9 : Every NCα-OS set represnts a NCi-OS set.

Proof **:** Let $\mathcal{F}$ be a NCα-OS in $(\mathcal{Q},\zeta)$, and ever sinceeach NCα-OS set forms a NCS-OS set. By proposition 3.7, $\mathcal{F}$ is a NCi-OS. □

The reverse of proposition 3.9 is not valid in common, as the next instance show.

Example 3.10 : In above example 3.8, we see that $\langle\{\varepsilon_1,\varepsilon_3\},\varphi,\varphi\rangle$ is a NCi-OS but is not NCα-OS.

Remark 3.11 : By proposition 3.3, proposition 3.7 and proposition 3.9, we have the following diagram:

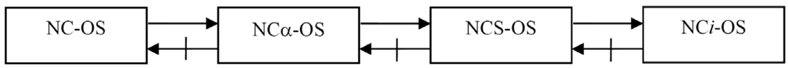

Fig. 3.1

Remark 3.12 : The intersection of NCi-OSs is not necessary to be NCi-OS.

Example 3.13 : Let $\mathcal{Q}=\{\eta_1,\eta_2,\eta_3,\eta_4\}$. Then $\zeta=\{\varphi_N,\langle\{\eta_1,\eta_2\},\varphi,\varphi\rangle,\mathcal{Q}_N\}$ is a NCTS. The collection of all NCi-OSs of $\mathcal{Q}$ is :

$$NCiO(\mathcal{Q})=\{\langle\{\eta_1\},\varphi,\varphi\rangle,\langle\{\eta_2\},\varphi,\varphi\rangle,\langle\{\eta_1,\eta_2\},\varphi,\varphi\rangle,\langle\{\eta_1,\eta_3\},\varphi,\varphi\rangle,\langle\{\eta_1,\eta_4\},\varphi,\varphi\rangle,\\ \langle\{\eta_2,\eta_3\},\varphi,\varphi\rangle,\langle\{\eta_2,\eta_4\},\varphi,\varphi\rangle,\langle\{\eta_2,\eta_3,\eta_4\},\varphi,\varphi\rangle,\varphi_N,\langle\{\eta_1,\eta_3,\eta_4\},\varphi,\varphi\rangle,\\ \mathcal{Q}_N,\langle\{\eta_1,\eta_2,\eta_4\},\varphi,\varphi\rangle,\langle\{\eta_1,\eta_2,\eta_3\},\varphi,\varphi\rangle\}.$$

We see that the set $\langle\{\eta_1,\eta_4\},\varphi,\varphi\rangle$, and the another set $\langle\{\eta_2,\eta_4\},\varphi,\varphi\rangle$, are NCi-OSs buttheir intersection is give us the not NCi-OS $\langle\{\eta_4\},\varphi,\varphi\rangle$.

Remark 3.14 : The union of NCi-OSs is not necessary to be NCi-OS.

Example 3.15 : Let $\mathcal{Q}=\{t_1,t_2,t_3,t_4\}$. Then

$$\zeta=\{\varphi_N,\langle\{t_1\},\varphi,\varphi\rangle,\langle\{t_1,t_2\},\varphi,\varphi\rangle,\langle\{t_1,t_3\},\varphi,\varphi\rangle,\langle\{t_1,t_4\},\varphi,\varphi\rangle,\langle\{t_1,t_2,t_3\},\varphi,\varphi\rangle,\\ \langle\{t_1,t_2,t_4\},\varphi,\varphi\rangle,\langle\{t_1,t_3,t_4\},\varphi,\varphi\rangle,\mathcal{Q}_N\}$$

is a NCTS.

The collection of all NCi-OSs of $\mathcal{Q}$ is:

$$NCiO(\mathcal{Q})=\{\langle\{t_1\},\varphi,\varphi\rangle,\langle\{t_2\},\varphi,\varphi\rangle,\langle\{t_3\},\varphi,\varphi\rangle,\langle\{t_4\},\varphi,\varphi\rangle,\varphi_N,\langle\{t_1,t_2\},\varphi,\varphi\rangle,\mathcal{Q}_N,\\ \langle\{t_1,t_3\},\varphi,\varphi\rangle,\langle\{t_1,t_4\},\varphi,\varphi\rangle,\langle\{t_2,t_3\},\varphi,\varphi\rangle,\langle\{t_2,t_4\},\varphi,\varphi\rangle,\langle\{t_1,t_2,t_3\},\varphi,\varphi\rangle,\\ \langle\{t_1,t_2,t_4\},\varphi,\varphi\rangle,\langle\{t_2,t_3,t_4\},\varphi,\varphi\rangle\}.$$

We see that the set $\langle\{t_2,t_3\},\varphi,\varphi\rangle$ and another set $\langle\{t_2,t_4\},\varphi,\varphi\rangle$ are NCi-OSs but their union $\langle\{t_2,t_3,t_4\},\varphi,\varphi\rangle$ is not NCi-OS.

4. Neutrosophic crisp ii -open sets

Definition 4.1 : A NC-subset $\mathcal{F}$ of a NCTS $\mathcal{Q}$ is declaredas a neutrosophic crisp inter-open set (in short NCint-OS) if there exist NC-OS $\mathcal{G} \in \zeta$ and $\mathcal{G} \neq \varphi_N, \mathcal{Q}_N$ such that $NC\text{-}\operatorname{int}(\mathcal{F}) = \mathcal{G}$. The set complement of a NCint-OS is named a neutrosophic crisp inter-closed set (in a few NCint-CS) in $\mathcal{Q}$. The collection of all NCint-OSs (resp. NCint-CSs) of $\mathcal{Q}$ is indicated by $NC\operatorname{int}O(\mathcal{Q})$ (resp. $NC\operatorname{int}C(\mathcal{Q})$).

Definition 4.2 : A NC-subset $\mathcal{F}$ of a NCTS $\mathcal{Q}$ is identifiedas a neutrosophic crisp ii-open set (briefly NCii-OS) if there exist NC-OS $\mathcal{G} \in \zeta$ such that $\mathcal{G} \neq \varphi_N, \mathcal{Q}_N$, $\mathcal{F} \subseteq NC\text{-}\operatorname{cl}(\mathcal{F} \cap \mathcal{G})$ and $NC\text{-}\operatorname{int}(\mathcal{F}) = \mathcal{G}$. The set complement of a NCii-OS is named a neutrosophic crisp ii-closed set (in short NCii-CS) in $\mathcal{Q}$. The collection of all NCii-OSs (resp. NCii-CSs) of $\mathcal{Q}$ is signified by $NC\text{ii}O(\mathcal{Q})$ (resp. $NC\text{ii}C(\mathcal{Q})$).

Example 4.3 : Let $\mathcal{Q} = \{q_1, q_2, q_3\}$. Then $\zeta = \{\langle\{q_2\}, \varphi, \varphi\rangle, \varphi_N, \mathcal{Q}_N, \langle\{q_2, q_3\}, \varphi, \varphi\rangle\}$ is a NCTS. Here

$NC\text{i}O(\mathcal{Q})$

$$= \{\langle\{q_2\}, \varphi, \varphi\rangle, \langle\{q_3\}, \varphi, \varphi\rangle, \langle\{q_1, q_2\}, \varphi, \varphi\rangle, \varphi_N, \langle\{q_1, q_3\}, \varphi, \varphi\rangle, \mathcal{Q}_N, \langle\{q_2, q_3\}, \varphi, \varphi\rangle\};$$

$$NC\text{int}O(\mathcal{Q}) = \{\langle\{q_2\}, \varphi, \varphi\rangle, \langle\{q_1, q_2\}, \varphi, \varphi\rangle, \varphi_N, \mathcal{Q}_N, \langle\{q_2, q_3\}, \varphi, \varphi\rangle\};$$

$$NC\text{ii}O(\mathcal{Q}) = \{\langle\{q_2\}, \varphi, \varphi\rangle, \langle\{q_1, q_2\}, \varphi, \varphi\rangle, \varphi_N, \mathcal{Q}_N, \langle\{q_2, q_3\}, \varphi, \varphi\rangle\}.$$

Proposition 4.4 : Each NC-OS is a NCii-OS.

Proof **:** Assume $\mathcal{V}$ represnts a NC-OS set in $\mathcal{Q}$. While $\mathcal{V} \subseteq NC\text{-}\operatorname{cl}(\mathcal{V} \cap \mathcal{V}) = NC\text{-}\operatorname{cl}(\mathcal{V})$, this notes $\mathcal{V}$ is a NCi-OS. Also, $\mathcal{V}$ is a NCint-OS because $NC\text{-}\operatorname{int}(\mathcal{V}) = \mathcal{V}$. Thus $\mathcal{V}$ is a NCii-OS. □

The reverse of proposition 4.4 is not valid in common, as the next instance show.

Example 4.5 : Let $\mathcal{Q} = \{q_1, q_2, q_3\}$. Then $\zeta = \{\langle\{q_2\}, \varphi, \varphi\rangle, \varphi_N, \mathcal{Q}_N, \langle\{q_2, q_3\}, \varphi, \varphi\rangle\}$ is a NCTS. Here

$NC\text{i}O(\mathcal{Q}) =$

$$\{\varphi_N, \langle\{q_2\}, \varphi, \varphi\rangle, \langle\{q_3\}, \varphi, \varphi\rangle, \langle\{q_1, q_2\}, \varphi, \varphi\rangle, \langle\{q_1, q_3\}, \varphi, \varphi\rangle, \langle\{q_2, q_3\}, \varphi, \varphi\rangle \mathcal{Q}_N\};$$

$$NC\text{int}O(\mathcal{Q}) = \{\langle\{q_2\},\varphi,\varphi\rangle,\langle\{q_1,q_2\},\varphi,\varphi\rangle,\varphi_N,\mathcal{Q}_N,\langle\{q_2,q_3\},\varphi,\varphi\rangle\};$$

$$NC\text{ii}O(\mathcal{Q}) = \{\varphi_N,\langle\{q_2,q_3\},\varphi,\varphi\rangle,\langle\{q_2\},\varphi,\varphi\rangle,\langle\{q_1,q_2\},\varphi,\varphi\rangle,\mathcal{Q}_N,\}.$$

Now, $\langle\{q_1,q_2\},\varphi,\varphi\rangle$ is a NCii-OS but not NC-OS.

Remark 4.6 : Every NCii-OS is a NCi-OS and NCint-OS. Further, every NCS-OS represnts a NCii-OS set. However, the reverse is not valid in common, as the nextinstance show.

Example 4.7 : Let $\mathcal{Q} = \{q_1,q_2,q_3\}$. Then $\zeta = \{\langle\{q_2,q_3\},\varphi,\varphi\rangle,\varphi_N,\mathcal{Q}_N,\langle\{q_1\},\varphi,\varphi\rangle\}$ is a NCTS. Here

$$NC\text{i}O(\mathcal{Q}) = \{\langle\{q_1\},\varphi,\varphi\rangle,\langle\{q_2\},\varphi,\varphi\rangle,\varphi_N,\langle\{q_3\},\varphi,\varphi\rangle,\mathcal{Q}_N,\langle\{q_2,q_3\},\varphi,\varphi\rangle\};$$

$$NC\,\text{int}\,O(\mathcal{Q}) = \{\langle\{q_1\},\varphi,\varphi\rangle,\varphi_N,\langle\{q_1,q_2\},\varphi,\varphi\rangle,\langle\{q_1,q_3\},\varphi,\varphi\rangle,\mathcal{Q}_N,\langle\{q_2,q_3\},\varphi,\varphi\rangle\};$$

$$NC\text{ii}O(\mathcal{Q}) = NCSO(\mathcal{Q}) = \{\langle\{q_2,q_3\},\varphi,\varphi\rangle,\varphi_N,\mathcal{Q}_N,\langle\{q_1\},\varphi,\varphi\rangle\}.$$

Then, $\langle\{q_2\},\varphi,\varphi\rangle$ is a NCi-OS but not NCii-OS. However $\langle\{q_1,q_2\},\varphi,\varphi\rangle$ is a NCint-OS but not NCii-OS.

Proposition 4.8 : Every NCα-OS is a NCii-OS.

***Proof* :** Let $\mathcal{Q}$ be NCTS, and $\mathcal{F} \subseteq \mathcal{Q}$ be NCα-OS. Since $\mathcal{F} \subseteq NC\text{-int}(NC\text{-cl}(NC\text{-int}(\mathcal{F}))) \subseteq NC\text{-cl}(NC\text{-int}(\mathcal{F}))$. Therefore $\mathcal{F}$ is a NCS-OS. Since there exists a NC-OS, say, $\mathcal{G} \neq \varphi_N,\mathcal{Q}_N$ satisfying $NC\text{-int}(\mathcal{F}) \subseteq \mathcal{G}$, this leads to $NC\text{-int}(\mathcal{F}) \subseteq \mathcal{G} \cap \mathcal{F}$. Therefore $\mathcal{F} \subseteq NC\text{-cl}(\mathcal{F} \cap \mathcal{G})$. Thus, $\mathcal{F}$ is a NCi-OS. We shall prove that $NC\text{-int}(\mathcal{F}) = \mathcal{G}$. Note that if $NC\text{-int}(\mathcal{F}) \neq \mathcal{G}$, for all $\mathcal{G} \in \zeta$, then $NC\text{-cl}(NC\text{-int}(\mathcal{F})) \neq NC\text{-cl}(\mathcal{G})$. From the above inclusions, we conclude that $\mathcal{F} \subseteq NC\text{-cl}(NC\text{-int}(\mathcal{F}) \cap \mathcal{F} \cap \mathcal{G})$. This implies that $\mathcal{F} \subsetneq NC\text{-cl}(\mathcal{G})$. That is a contradiction. Therefore, $\mathcal{F}$ is a NCii-OS. □

The reverse of proposition 4.8 is not valid in common, as the nextinstance show.

Example 4.9 : Let $\mathcal{Q} = \{t_1,t_2,t_3,t_4\}$. Then

$$\zeta = \{\varphi_N,\langle\{t_1\},\varphi,\varphi\rangle,\langle\{t_2,t_3,t_4\},\varphi,\varphi\rangle,\mathcal{Q}_N\}$$

is a NCTS. Here

$$NC\alpha O(\mathcal{Q}) = \{\varphi_N, \langle\{t_1\}, \varphi, \varphi\rangle, \langle\{t_2, t_3, t_4\}, \varphi, \varphi\rangle, \mathcal{Q}_N\};$$

$$NC\text{ii}O(\mathcal{Q}) = \{\varphi_N, \langle\{t_1\}, \varphi, \varphi\rangle, \langle\{t_2\}, \varphi, \varphi\rangle, \langle\{t_3\}, \varphi, \varphi\rangle, \langle\{t_4\}, \varphi, \varphi\rangle, \langle\{t_2, t_3\}, \varphi, \varphi\rangle,$$

$$\langle\{t_2, t_4\}, \varphi, \varphi\rangle, \langle\{t_3, t_4\}, \varphi, \varphi\rangle, \langle\{t_2, t_3, t_4\}, \varphi, \varphi\rangle, \mathcal{Q}_N\}.$$

Now, $\langle\{t_3\}, \varphi, \varphi\rangle$ is a NCii-OS but not NCα-OS.

Proposition 4.10 : Every NCα-OS is a NCint-OS.

***Proof* :** Clear. □

The reverse of proposition 4.10 is not valid in common, as the next example show.

Example 4.11 : Let $\mathcal{Q} = \{q_1, q_2, q_3\}$. Then

$$\zeta = \{\langle\{q_2, q_3\}, \varphi, \varphi\rangle, \varphi_N, \mathcal{Q}_N, \langle\{q_1\}, \varphi, \varphi\rangle\}$$

is a NCTS. Here

$$NC\alpha O(\mathcal{Q}) = \{\langle\{q_2, q_3\}, \varphi, \varphi\rangle, \varphi_N, \mathcal{Q}_N, \langle\{q_1\}, \varphi, \varphi\rangle\};$$

$$NC\,\text{int}\,O(\mathcal{Q}) = \{\varphi_N, \langle\{q_1\}, \varphi, \varphi\rangle, \langle\{q_1, q_2\}, \varphi, \varphi\rangle, \langle\{q_1, q_3\}, \varphi, \varphi\rangle, \langle\{q_2, q_3\}, \varphi, \varphi\rangle, \mathcal{Q}_N\};$$

Now, $\langle\{q_1, q_2\}, \varphi, \varphi\rangle$ is a NCint-OS but not NCα-OS.

Theorem 4.12 : If $\mathcal{Q}$ is a NCTS, then $(\mathcal{Q}, NC\text{ii}O(\mathcal{Q}))$ is a NCTS.

***Proof* :** Clear. □

Example4.13: Let $\mathcal{Q} = \{q_1, q_2, q_3\}$. Then $\zeta = \{\langle\{q_2\}, \varphi, \varphi\rangle, \varphi_N, \mathcal{Q}_N, \langle\{q_2, q_3\}, \varphi, \varphi\rangle\}$ is a NCTS. Here

$$NC\text{ii}O(\mathcal{Q}) = \{\langle\{q_1, q_2\}, \varphi, \varphi\rangle, \varphi_N, \langle\{q_2, q_3\}, \varphi, \varphi\rangle, \mathcal{Q}_N, \langle\{q_2\}, \varphi, \varphi\rangle\};$$

Note that $\mathcal{Q}$ is a NCTS and $(\mathcal{Q}, NC\text{ii}O(\mathcal{Q}))$ is a NCTS.

Remark 4.14 : The following diagram shows that the relationships of NCii-OSs with other NC-sets.

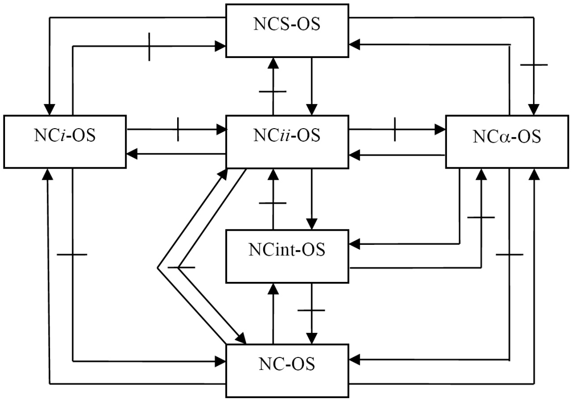

Fig. 4.1

5. Conclusion

We reveal new ideas known as NC i-OSs, NC inter-OSs and NC ii-OSs by taking a broad view of neutrosophic crisp open set in its related topological space. Several representations of these notions are reviewed, and the connections of these notions with different other ideas of neutrosophic crisp open sets are shown.The concepts can be manipulated to develop some neutrosophic crisp separation axioms.

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