Journal of Interdisciplinary Mathematics

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/tjim20

Graph strongly by *Qpre*-closed set in topological spaces

Zinah Taha Abdulqader, Sarah Haider Khalil & Mustafa Hasan Hadi

To cite this article: Zinah Taha Abdulqader, Sarah Haider Khalil & Mustafa Hasan Hadi (2022) Graph strongly by *Qpre*-closed set in topological spaces, Journal of Interdisciplinary Mathematics, 25:6, 1829-1834, DOI: 10.1080/09720502.2022.2052573

To link to this article: https://doi.org/10.1080/09720502.2022.2052573

Published online: 26 Sep 2022.

Journal of Interdisciplinary Mathematics
ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online)
Vol. 25 (2022), No. 6, pp. 1829–1834
DOI : 10.1080/09720502.2022.2052573

Graph strongly by *Qpre*-closed set in topological spaces

Zinah Taha Abdulqader [§]
Department of Mathematics
College of Education for Woman
University of Tikrit
Tikrit
Iraq

Sarah Haider Khalil [†]
General Directorate of Education
Hillah
Iraq

Mustafa Hasan Hadi [*]
Department of Mathematics
College of Education for Pure Sciences
University of Babylon
Hillah
Iraq

Abstract

In this paper, we summed a new type of graph called graph strongly *Qpre*-closed based on the functions resulting from cross-protected that use the sets. We did not show new types of relations, theorems, and illustrative examples about this new concept, which is considered one of the practical applications in the topological space and which is of great importance in our daily life.

Subject Classification: *54A25, 54A35, 54G10.*

Keywords: *Graph strongly, Separation axioms, Qpre–graph, Qpre-continuous.*

[§] *E-mail:* ztaha@tu.edu.iq
[†] *E-mail:* Sh7459281@gmail.com
[*] *E-mail:* pure.mustafa.hassan@uobabylon.edu.iq (Corresponding Author)

1. Introduction

One of the important topics in topology that has not been talked about much is graph, which is based on topological spaces using functions, which are necessary and modern concepts in modern topology. To begin with, let us recall some of the basic concepts in the construction of this research. Let $Q \subseteq X$ be *Qpre*-closed set if $Cl_g(\text{Int}(Q)) \subseteq Q$. The complement is called *Qpre*-open, we will symbolize $Qpre\,O(X,x)$ and $Qpre\,C(X,x)$ is that family of all *Qpre*-open sets and *Qpre*-closed are respectively [1] , this definition is already known as Hadi, M.H. and Al-Yaseen, M.A.A. K. , the first to know strongly closed graph [2] by Herrington, L.L, this definition has also been generalized by the use of other weak sets, namely *pre*-open her name graph strongly *pre*-closed [3] by Noiri , T. and Jafari, S. , in [4] Abdulqader, Z.T. and Hadi, M.H. has been defined *Qpre*-closed graph. Finally, we define a new type of graph by a set of *Qpre*-closed [1], which is more powerful and presents proofs and present proofs and arguments on this important and fundamental topic.

We will also link this modern concept to separation axioms and provide proof about this thing through which this definition can link to other concepts. A question that may arise here is what is the difference between the term graph in the theory of graph and this term. The difference between these two definitions depends on the building functions that used the cross-protection, which was initially built using sets between them. Still, the usual definition of a graph is based on the point. This definition can be used in references [5, 6] and a generalization to the set in the papers [7,8] that was interested in studying the types of approximation which can be developed according to the definition in our research work, for more information about it, please see [9,10].

2. Graph Strongly *Qpre*-Closed

Definition 2.1 : A function $f: X \to Y$ has a graph strongly *Qpre*-closed $\forall\ (x\ ,y) \in\ (X \times Y) \setminus G(f)$, $\exists A \in\ Qpre\,O(X,x)$ and $B \in O(Y,y)$ such that $(A \times CL(B)) \cap G(f) = \varnothing$.

Let us now review examples, important proofs, and notes.

Remark 2.2 :

(1) Any graph strongly *Qpre*-closed is *Qpre*-closed. However, the opposite is not true. (see Example 2.4) in [4] , where $\{¥_2\} \in QO(Y,\ ¥)$,

but $\{¥_2\} \notin O(Y)$, then $G(f)$ is *Qpre*-closed but not strongly *Qpre*-closed .

(2) Any graph strongly *pre*-closed (resp. graph strongly closed) is graph strongly *Qpre*–closed, However, the opposite is not true.

We can summarize the relationships among the graph strongly closed by the following diagram.

Graph strongly closed $\rightarrow$ Graph strongly *pre*-closed $\rightarrow$ Graph strongly *Qpre*-closed.

Lemma 2.3 : *A function* $f: X \rightarrow Y$ *is a graph strongly Qpre-closed* iff $\forall (x, y) \in (X \times Y) \setminus G(f)$, $\exists A \in QpreO(X, x)$ *and* $B \in O(Y, y)$ *s.t.* $f(A) \cap CL(B) = \varnothing$.

***Proof* :** Very clear from the diagram.

Theorem 2.4 : *A function* $f: X \rightarrow Y$ *is a graph strongly Qpre–closed if* $\forall (x, y) \in (X \times Y) \setminus G(f)$, $\exists A \in QpreO(X, x)$ *and* $B \in preO(Y, y)$ *s.t.* $(A \times Cl_{pre}(B)) \cap (f) = \varnothing$.

Lemma 2.5 : *A function* $f: X \rightarrow Y$ *is a graph strongly Qpre–closed if* $\forall (x, y) \in (X \times Y) G(f)$, $\exists A \in QpreO(X, x)$, $B \in preO(Y, y)$ *s.t.* $f(A) \cap Cl_{pre}(B) = \varnothing$.

Theorem 2.6 : *Let* $f: X \rightarrow Y$ *is graph strongly Qpre-closed, so* $\forall x_0 \in X$, $f(x) = \bigcap \{Cl_{pre}(f(A)) : A \in QpreO(X, x_0)\}$.

***Proof* :** Assume the Theorem is incorrect. Then $\exists y_0 \neq f(x_0)$ s.t. $y_0 \in \bigcap \{Cl_{pre}(f(A) : A \in QpreO(X, x_0)$. Then $y_0 \in Cl_{pre}(f(A))$ (for any $A \in QpreO(X, x_0)$. Therefore $B \cap f(A)$ is empty $\forall B \in preO(Y, y_0)$. This is, the mean $f(A) \subset B \subset f(A) \cap Cl_{pre}(B) \neq \varnothing$, this contradicts with the hypothesis that the function is *Qpre*–closed graph. Therefore, the theorem is true.

Theorem 2.7 : *Let* $f: X \rightarrow Y$ *is Qpre–continuous and Y is* T_2*-space. So* $G(f)$ *graph strongly Qpre–closed.*

Proof : Suppose that $(x_0, y_0) \in (X \times Y) \setminus G(f)$. Since Y is T_2-space, then $\exists B \in O(Y, y_0)$ s.t. $f(x_0) \notin CL(B)$. But $CL(B)$ is a closed set. Now $Y \setminus CL(B) \in O(Y, f(x_0))$, so $\exists A \in QpreO(X, x_0)$, s.t. $f(A) \subseteq Y \setminus CL(B)$. And therefore $f(A) \cap CL(B) = \varnothing$ and so $G(f)$ is graph strongly *Qpre*-closed.

Theorem 2.8 : $f: X \rightarrow Y$ *is onto,* $G(f)$ *strongly Qpre-closed. So Y* $T\text{-}_1$ *space.*

Proof **:** Suppose that c and d point of Y s.t. $c \neq d$. Since f is onto this mean $f(x_1) = d$, $\exists x_1 \in X$ and $(x_1, c) \in (X \times Y) \setminus G(f)$. Since the graph strongly *Qpre*–closed of $G(f)$ provides $F_1 \in QpreO(X, x_1)$ and $G_1 \in O(Y, c)$ s.t. $f(F_1) \cap CL(G_1) = \varnothing$. Now, $x_1 \in F_1$, this implies $f(x_1) = d \in f(F_1)$. This means that $f(F_1) \cap CL(G_1) = \varnothing$, warranty that $d \notin G_1$. Also from the subjectivity of f we obtain $x_2 \in X$ s.t. $f(x_2) = c$. Now, $(x_2, d) \in (X \times Y) \setminus G(f)$ and the strongly *Qpre*-closed of $G(f)$ provides $F_2 \in QpreO(X, x_2)$, $G_2 \in O(Y, d)$ s.t. $f(F_2) \cap CL(G_2) = \varnothing$. Now, $x_2 \in F_2$ this implies that $f(x_2) = c \in f(F_2)$, then $c \notin G_2$ therefore, we obtain sets $G_1, G_2 \in O(Y)$ s.t. $c \in G_1$ but $d \notin G_1$ while $d \in G_2$ but $c \notin G_2$, therefore Y is T_1 space.

Proposition 2.9 : Let $f : X \to Y$ is onto, $G(f)$ graph strongly *Qpre*– closed. So Y is pre – T_1.

Proof **:** Same idea as proof Theorem (2.4).

Proposition 2.10 : A function $f : X \to Y$ is onto and $G(f)$ graph strongly *Qpre*-closed. So Y *Hpre*-T_1 space**.**

Proof **:** Same idea as proof Theorem (2.4).

Corollary 2.11 : *Every one-to-one, onto $f : X \to Y$, $G(f)$ Hpre-closed. So X and Y are Hpre-T_1.*

Proof **:** Direct from Proposition (2.10).

Theorem 2.12 : *Let $f : X \to Y$ is one-to-one and $G(f)$ graph strongly Qpre-closed. So X is Hpre-$T_{1.}$*

Proof **:** Let a and b be any point of X and $x \neq y$. Since f is one-to-one this means $f(a) \neq f(b)$ when of obtains that $(a, f(b)) \in (X \times Y) \setminus G(f)$. The graph strongly *Qpre*-closedness of $G(f)$ provides $F_1 \in HpreO(X, a), G_1 \in O(Y, b)$ s.t. $f(F_1) \cap CL(G_1) = \varnothing$. So $f(b) \notin f(F_1)$ means $b \notin F_1$. Also $(b, f(a)) \in (X \times Y) \setminus G(f)$ and graph strongly *Qpre*-closedness of $G(f)$ then $F_2 \in QpreO(X, b), G_2 \in O(Y, f(a))$ with $(F_2) \cap CL(G_2) = \varnothing$, warranty that $f(a) \notin f(F_2)$ and therefore $a \notin F_2$. So F_1 and $F_2 \in HpreO(X)$, s.t. $a \in F_1$ but $b \notin F_1$ while $b \in F_2$ but $a \notin F_2$. Hence X is *Hpre*-T_1.

Proposition 2.13 : Let $f : X \to Y$ is one-to-one, onto and $G(f)$ graph strongly *Qpre*-closed. So are X and Y *Hpre*-T_1.

Proof **:** Direct by Proposition (2.10) and Corollary (2.11).

Theorem 2.14 : *Let $f: X \to Y$ is onto and $G(f)$ graph strongly Qpre-closed. So Y is T_2.*

***Proof* :** Suppose that a and b in Y s.t. $x \neq y$. Since f is onto this means, $f(x_1) = a$, $\exists\, x_1 \in X$ and $(x_1, b) \in (X \times Y) \setminus G(f)$. Graph strongly *Qpre*-closedness of $G(f)$ provides $A \in QpreO(X, x_1)$ and $B \in O(Y, b)$ s.t. $f(A) \bigcap CL(B) = \varnothing$. Now $x_1 \in A$ this implies $f(x_1) = a \in f(A)$. This means that $f(A) \bigcap CL(B) = \varnothing$ warrant that, $a \notin (B)$. Then there exists $M \in O(Y, a)$ such that $M \bigcap B = \varnothing$. So Y is T_2.

Proposition 2.15 : A function $f: X \to Y$ is onto and $G(f)$ graph strongly *Qpre*–closed. So Y is $pre-T_2$ space.

***Proof* :** Direct by Theorem (2.14).

Proposition 2.16: Let $f: X \to Y$ is onto and $G(f)$ graph strongly *Qpre* –closed. So Y is $Hpre - T_2$.

***Proof* :** Direct by Theorem (2.14).

3. Conclusion

The definition of graph strongly *Qpre* - closed, generalization to new concepts, and its association with competence can be used in references [11,12] as well as it can be used in ideal topology which can be found in [4,10,13].

References

[1] Hadi, M.H. and Al-Yaseen, M.A.A.K., Study of Hpre -open sets in topological spaces, AIP Conference Proceedings 2292, (2020).

[2] Herrington, L.L. and Long p.p., Characterizations of H-closed spaces, *proc. Amer. Math.Soc.,* vol. 48, pp. 469-75, (1975).

[3] Noiri , T., Jafari , S., Latif, R.M. and Caldas, M., Characterization of function with strongly pre-closed graphs, Sci. stud. Res. Ser. Math. In form, vol. 19, pp. 49-58, (2009).

[4] Hadi, M.H., AL-Yaseen, M.A.A.K. and Al-Swidi, L.A., Forms weakly continuity using weak ω–open sets, *Journal of Interdisciplinary Mathematics,* 24(5), pp. 1141-1144, (2021).

[5] Cammaroto, F. andNoiri, T., A note on weakly-compact spaces, Indian *J. pure appl. Math.*, 16(12), pp. 1472-1477, (1985).

[6] Noiri, T., Al-Omari, A. A. and Noorani, M. S. M., Weak forms of ω-open sets and decomposition of continuitym E.J.P.A.M., 2(1), pp.73-84, (2009).

[7] Maheshwari, S. N. and Prasad, R., Some new separation axioms, Ann. Soc. Sci. Bruxelles 89, pp. 395-402, (1975).

[8] Kar, A. and Bhattacharyya, P., Some weak separation axioms, *Bull. Calcutta Math.Soc.* 82, pp. 415-422, (1990).

[9] Jankovic, D. and Hamlett,T. R., New topologies from old via ideals, *The American Mathematical Monthly,* vol. 97, pp. 295–310, (1990).

[10] Mashhour, A. S., Abd El- Monsef, M. E. and El- Deeb, S. N., On pre-continuous and weak pre-continuous functions, Proc. Math. Phys. Soc. Egypt 51, pp. 47-53, (1982).

[11] Wei Gao and Mohammad Reza Farahani, The Zagreb topological indices for a type of Benzenoid systems jagged-rectangle, *Journal of Interdisciplinary Mathematics,* 20(5), PP. 1341-1348, (2017).

[12] Al-Omari, A. and Noorani, M. S. M., Regular generalized ω-closed sets, *Internat. J. Math. Math.Sci.,* (2007).

[13] Al-Zoubi, K. and Al-Nashef, B., The topology of ω-open subsets, Al-Manareh, 9(2), pp. 169-179, (2003).

Received October 2021

Revised December 2021