Free vibration and Buckling Behavior of Tapered Beam by Finite Element Method

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Abstract

This paper deals with free vibration and buckling behavior of non-uniform Euler-Bernoulli beam under variation of tapered parameter and degree of flexural bending by using Finite Element Method and linked with Matlab Program. The results obtained were compared with those results given in the literatures and it is found that the natural frequency and buckling load decrease with increasing the tapered parameter and degree of flexural stiffness of tapered beams.

الخلاصة

هذا البحث يهتم بدراسة الاهتزاز الحر وسلوك الانبعاج لعتبة اويلر- برنولي مستدقة غير منتظمة بتغير معامـــل الاســـتدقاق ودرجة الأس لجساءة الانحناء باستخدام العناصر المحددة و برنامج الماتلاب. النتائج التي تم الحصول عليها قورنـــت مـــع نتـــائج مصادر أخرى، ووجد بان التردد وحمل الانبعاج يقل بزيادة معامل الاستدقاق ودرجة الأس لجساءة الانحناء للعتبة المستدقة.

Nomenclature

Symbols	Meaning
m_x	mass distribution per unit length
EI_x	flexural stiffness per unit length
w_x	displacement in z direction
q^T	deflection vector
N_i	interpolation factors
$N_i(\xi)$	shape function
$\xi = \frac{x}{l}$	non –dimensional length of beam element
U	potential energy
K_e	elastic stiffness
T	kinetic energy
m_e	mass matrix
V	strain energy
λ	a critical bucking load
P_o	value of axial compression force
K_g	geometrical stiffness matrix
r_m	tapered beam for mass
r_s	tapered beam for stiffness
ß	non- dimensional natural frequency
n	Number of element
i	Number of node
l	length of beam
ω_n	natural frequency

1-Introduction

Tapered members are widely used in the modern construction industry, because of their (i) structural efficiency, which in turn may lead to significant material savings, (ii) ability to meet architectural and functional requirements and (iii) competitive fabrication costs. However, a designer can only take full advantage of the benefits of beam tapering provided that he is equipped with reliable and efficient methods of analysis, which (i) lead to accurate predictions of the tapered member structural behavior and, at the same time, (ii) do not involve a computer effort prohibitive for routine applications.. The strength of laterally unrestrained beams is frequently governed by the lateral buckling (or flexural-tensional buckling) failure.

The vibration and bulking problems of non-uniform beams have been extensively studied by several investigators. Several cases of tapered beams with different end conditions were obtained by ***Mabie and Rogers*** ,while ***Laura*** treats various cases of non-uniform beams with different conditions of end restraints. A direct solution for the transverse vibration of Euler–Bernoulli wedge and cone beams was obtained by ***Naguleswaran***, while ***Abrate*** obtains the exact solution for the vibration of non-uniform rods and beams. ***Lee et al.*** were studied the analysis of non-uniform beam vibration by a green function method in the Laplace transform domain. ***Brown*** was studied the lateral buckling load of a tapered beam by finite difference analysis. In this method, the effect of tapering could not be completely taken into account in the expressions of nonlinear strains, which may lead to incorrect lateral buckling loads. ***Ronagh et al.*** was found the errors in lateral buckling loads caused by this method cannot be eliminated merely using fine mesh configuration in the finite element analysis. ***Ronagh et al***. and recently ***Andrade and Camotim*** were investigated the lateral buckling of tapered beams employing the FE method, based on their total potentials presented.

Si Yuan et. were studied the exact dynamic stiffness method for non-uniform Timoshenko beam vibration and Bernoulli Euler column bulking while ***Jung et.*** deals with free vibration problems of non-uniform Euler-Bernoulli beam under various supporting conditions. ***Zang Lei.*** was presented a new theory for the lateral buckling of wet-tapered I-beam while ***Vaidotas Sapalau*** studies a theoretical and numerical analysis of tapered beam- columns subjected to a bending moment and axial force. ***Aniosio A.*** was studied the lateral torsional buckling of singly symmetric web-tapered thin –walled I beam by using FEM.

In this study a finite element method is introduced to solve the vibration and buckling of non-uniform beam with clamped-free boundary condition to obtain natural frequencies and buckling loads.

2- Theoretical Analysis

A schematic of mass m(x) and flexural stiffness per unit length EI(x) is shown in fig(1), W(x) is the displacement in Z direction. In the present work, is a considered Euler-Bernoulli beams for analysis of out-of-plane bending vibration. The following function is used in the analysis to represent deflection which is given by ***Si Yuan***

$$W(x) = a_1 + a_2 x + a_3 x^2 + a_4 x^3 \qquad (1)$$

The deflection vector of the elemental finite element is

$$q^T = [V_1 \quad \theta_1 \quad V_2 \quad \theta_2] \qquad (2)$$

After application of the boundary conditions of clamped- free to calculate the interpolation functions (N_1, N_2, N_3, N_4) to describe the distribution of displacement. Eq.(1) can be written as in finite element formulation,

$$W(x) = N_1 V_1 + N_2 \theta_1 + N_3 V_2 + N_4 \theta_2 \qquad (3)$$

Where

$$N_1(\xi) = 1 - 3\xi^2 + 2\xi^3$$

$N_2(\xi) = L(\xi - 2\xi^2 + \xi^3)$ shape factors

$N_3(\xi) = 3\xi^2 - 2\xi^3$

$N_4(\xi) = L(\xi^3 - \xi^2)$

Where, $\xi = \frac{x}{L}$, is the non-dimensional length of beam element (element coordinates).

The potential energy of the beam element is given by ***Si Yuan*** :

$$U = \frac{1}{2}\int_0^L EI(x)[(W''(x,t))^2]dx \quad (4)$$

Eq.(4) can be written in matrix form as

$$U = \frac{1}{2}q^T k_e q \quad (5)$$

Where, k_e, elastic stiffness matrix.

$$k_e = \int_0^L EI(x)N_i'' N_j'' dx \quad (6)$$

Where, $N_i'' = \frac{d^2 N_i}{d\xi^2}$

The kinetic energy of the beam element is

$$T = \frac{1}{2}\int_0^L m(x)[W^{\bullet}(x)]^2 dx \quad (7)$$

Eq.(7) can be written in matrix form as

$$T = \frac{1}{2}q^T m_e q \quad (8)$$

Where, m_e, mass matrix

$$m_e = \int_0^L m(x)N_i N_j dx \quad (9)$$

The strain energy V denotes the work down by a critical load (λP_o)
Given by the equation

$$V = \int_0^L \lambda p_0 [W']^2 dx \quad (10)$$

Where p_0 is a value of axial compression force.
Eq.(10) can be written in matrix form as

$$V = \frac{1}{2}q^T k_g q \quad (11)$$

Where, k_g, geometrical stiffness matrix

$$k_g = \int_0^L \lambda P_o N_i' N_j' dx \quad (12)$$

Where, $N_i' = \frac{d\ N_i}{d\xi}$

Mass and stiffness matrices of each beam element are used to form global mass and stiffness matrices. The dynamic response of a beam for a conservative system can be formulated by means of Lagrange's equation of motion in which the external forces are expressed in terms of time- dependent potentials, and then performing the required operations the entire system leads to the governing matrix equation of motion:

$$Mq^{\bullet\bullet} + [K_e - \lambda P_o K_e]q = 0 \qquad (13)$$

The above equation represents the solution of two relate problems, they are given by ***Si Yann***
:-

1- Free vibration $\qquad [K_e - \beta^2 M]q = 0 \qquad (14)$

2- Buckling behavior $\qquad [K_e - \lambda P_o K_g]q = 0 \qquad (15)$

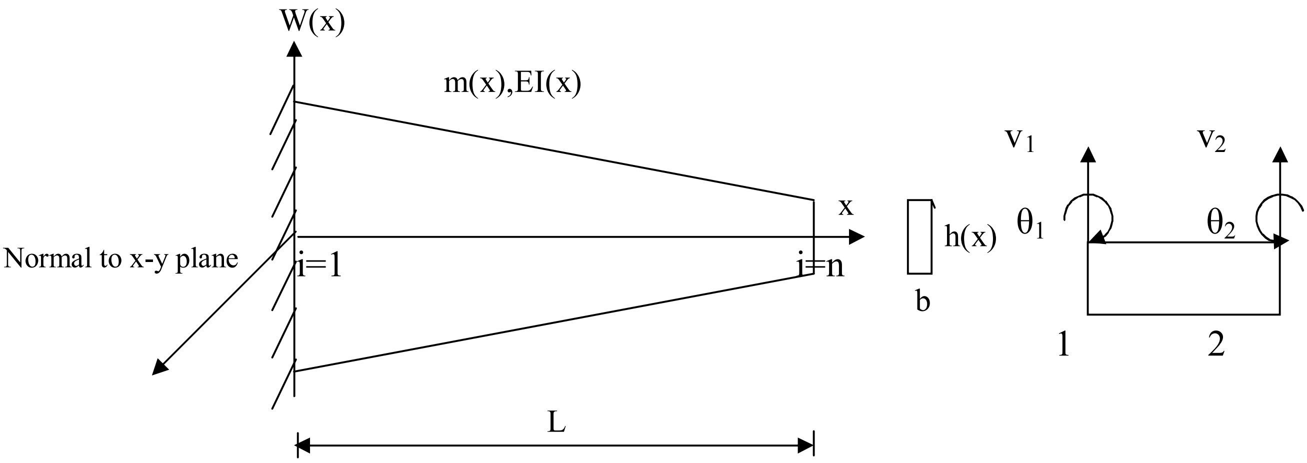

Fig (1)A: Cantilevered tapered beam with rectangular Cross-section

B: element of beam 2 nodes 4- DOF

3- Results and Discussion

Three case of mass and stiffness distribution were studied. In the first case linear mass and linear stiffness distribution, the second the mass is linear and stiffness is 2nd order. And third, the mass is linear and stiffness is 4th order.

Case(1): [Linear mass, Linear stiffness]

$m(x) = m_o(1 - r_m\xi)$

$EI(x) = EI_o(1 - r_s\xi)$

Case(2):[Linear mass, second degree stiffness]

$m(x) = m_o(1 - r_m\xi)$

$EI(x) = EI_o(1 - r_s\xi)^{i=2}$

Case(3):[Linear mass, forth degree stiffness]

$m(x) = m_o(1 - r_m\xi)$

$EI(x) = EI_o(1 - r_s\xi)^{i=4}$

Where (m_o) parameter for mass distribution per length ,at x=0, (EI_o) parameter for stiffness distribution of flexural rigidity**,** at x=0, and (r_m), (r_s)are the tapered beam for mass and stiffness and (i) is super index refer to the order of non linearity for both mass and stiffness distribution , respectively.

For all cases, value of (r_m) and (r_s) are equals and vary from ($0 \rightarrow 1$) step 0.25 to obtained its effect on the natural frequency and corresponding buckling loads for five modes only.

Table (1), (2) and (3) show an important comparison of non-dimensional natural frequency (ß) and critical buckling load of a uniform cantilever beam with results from **Jurg** et al. **Si Yuan** et al. and **Zhang Lei.**

Table:(1) Comparisons of non-dimensional natural frequency (β) of a uniform cantilever beam.

Mode No	**Present work**	**Jung**	**Si Yuan**
1	3.5155	3.5160	3.5161
2	22.0336	22.0345	22.0345
3	61.6963	61.6972	61.6972
4	120.9018	-------	120.9017
5	199.8575	------	199.8573

Table: (2) Comparison of critical buckling load of a uniform cantilever beam.(λ)

Mode No	Present work	Zhang Lei
1	2.4673	2.4682
2	22.2057	22.2055
3	61.6837	61.6835
4	120.8966	120.8965
5	199.8592	199.8590

Table(3): Comparison of non-dimensional natural frequency and critical buckling load of tapered cantilever beam.

Tapered Parameter	Mode No	$(\text{Non-dimensional Natural Frequency})^{1/2}$		$(\text{Critical Buckling Loads})^{1/2}$	
		Present work	Jung	Present work	Si Yuan
$r_m = 0.8$, $r_s = 0.95$	1	1.6768	1.6742	0.9712	0.9705
	2	4.1978	4.1969	2.1937	2.9131
	3	7.0244	7.0232	4.8562	4.8557
	4	9.8332	9.8314	6.7987	6.7982
	5	12.6428	12.6422	8.7412	8.7408
$r_m = 0.5$, $r_s = 0.5$	1	1.8585	1.8580	1.3352	1.3347
	2	4.6525	4.6523	4.0058	4.0050
	3	7.7853	7.7847	6.6764	6.6761
	4	10.8998	10.8990	9.3470	9.3465
	5	14.0122	-------	12.0176	12.0171

All the frequencies and buckling load predicted agree very well with the published results.

Figs (2,3), (4,5) and (6,7) present the non dimensional natural frequency and critical buckling loads for the first , second and third case respectively. From the first sight it was observed that the tapered ratio has a great effect on the vibration characteristics of the beam.

In general, it was noted that the natural frequency and corresponding critical buckling loads decrease with increasing in the taper ratio (r_m) and (r_s) for all modes, its magnitude are different from mode to other.

In other words, natural frequency of the 2nd mode (as example) for different values of tapered ratio are smaller than that corresponding for 3rd , 4th ,and 5th modes, respectively.

The band width of effectively is increased with increasing the modes number due to the increasing in the natural frequency and buckling loads when the mode number is increased also.

In addition, since the natural frequency is a structured property (means that its value is the same at each point in the structure), therefore, its value is largely depends on the effective stiffness and mass at same point. This leads to that natural frequency in the first case is greater than that corresponding to the second and third case, respectively. due to effect of super index (i) which reduces the stiffness contribution to obtain the frequency.

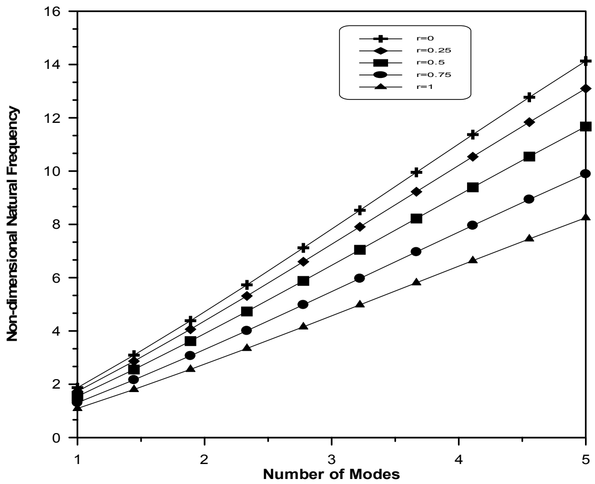

Fig (2): Variation of non-dimensional frequency with tapered parameter for case (1)

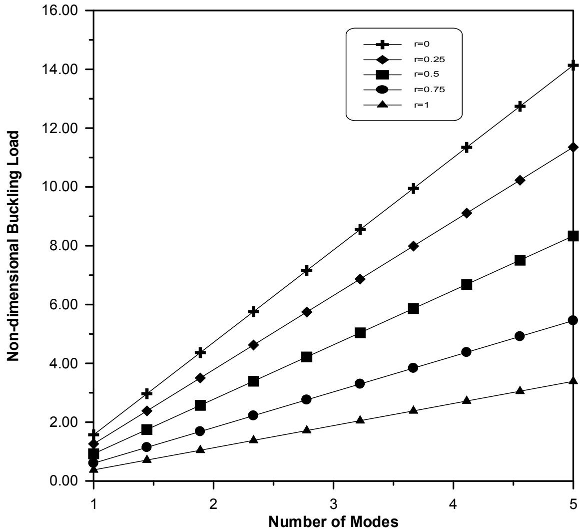

Fig (3): Variation of critical buckling load with tapered parameter for case (1)

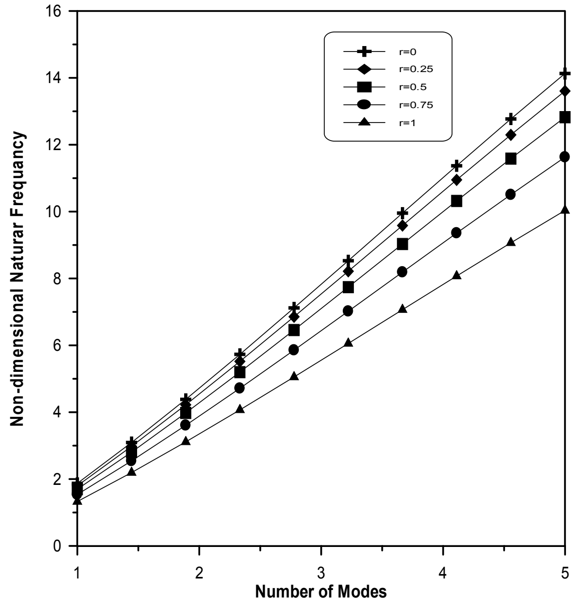

Fig (4): Variation of non-dimensional frequency with tapered parameter for case (2)

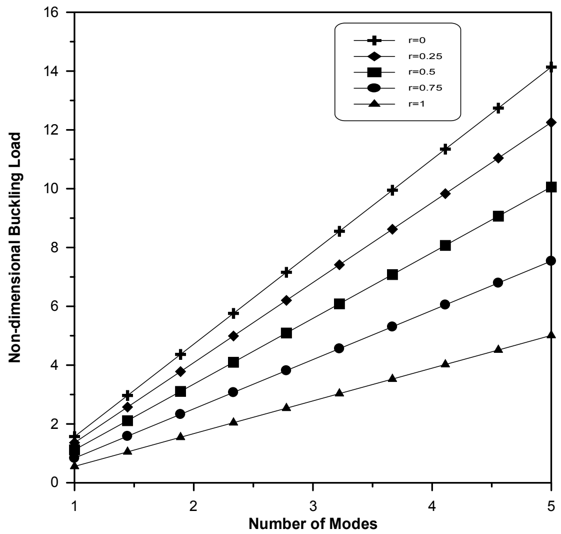

Fig (5): Variation of critical buckling load with tapered parameter for case (2)

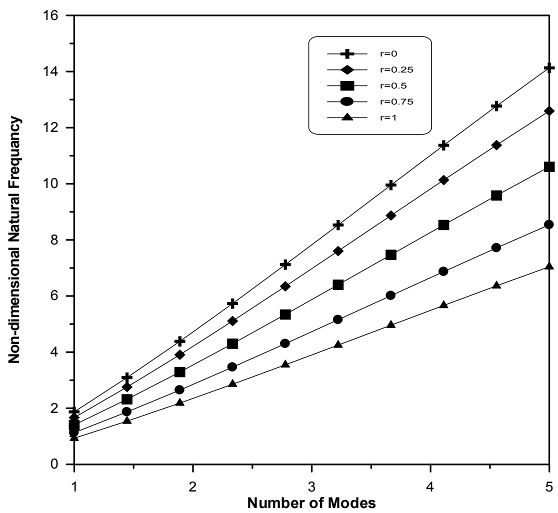

Fig (6): Variation of non-dimensional frequency with tapered parameter for case (3)

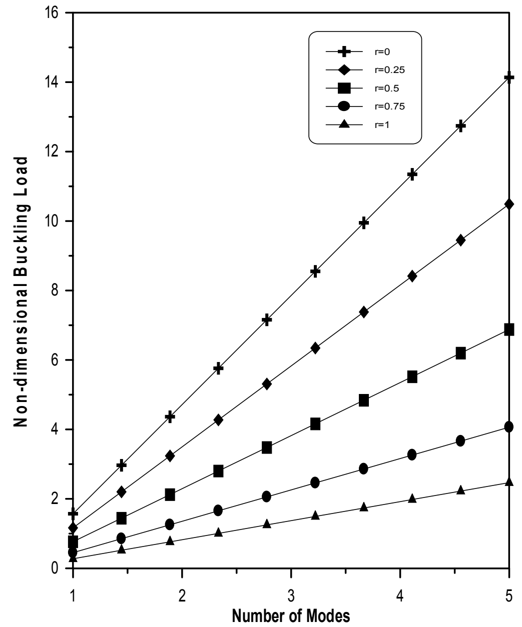

Fig (7): Variation of critical buckling load with tapered parameter for case (3)

4-Conclusions

In this paper the finite element method has been further extended to vibration and buckling of non-uniform Euler-Bernoulli beam from the results, some of the conclusions can be drawn as below

1- The non-dimensional frequency and critical buckling load predication obtained for the uniform and tapered beams are compared with the corresponding values mentored in the literature and found to agree very well.
2- The natural frequency and buckling load decrease with increasing of the tapered parameters.
3- The degree of the flexural load effect on the natural frequency and buckling load was decreased with the increasing in the degree of flexural bending stiffness.

5-References

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