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Abstract

The research is based on two basic principles: soft local function and soft Ψ-operation to construct the new concepts of density called *Ds*-dense and *Dc*-dense. Through these concepts, we defined a new type of separation axiom, which we called soft ST_1-space.

Subject Classification: *54H30, 54C65, 54B30.*

Keywords: *Soft set, Local function, Dense set, Soft complement, Locally dense and* D_c*-dense.*

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1. Introduction

The soft sets that the world has known Molodtsov, in 1999 [1] had a wide role in the applied scientific fields, which are considered more general than the fuzzy sets known to the world Zahah. In 1965 [2], it gave solutions to many scientific and engineering problems that the fuzzy sets were unable to solve perfectly. Simply the soft set is subset of the set $E_x P(X)$, where X is the universal set and E the set of all parameters of X. Many results were presented through previous studies on the soft topological space [3, 4] and on soft ideal topological spaces and ideal topological space [5,6,7]. Research is based on the soft local function defined by Kharal [4] in 2011 as follows, $(K_E)^* = \tilde{U}\{F_e^x; \forall U_E$ soft open set containing F_e^x such that $U_E \tilde{\cap} K_E \in \tilde{J}_E\}$, where $\tilde{J}_E$ is soft ideal also based on the soft $\Psi-$operator defined by Luay and Ali Abdulsada [8] in 2020 as follows $\Psi(K_E) = \tilde{\chi} - (\tilde{\chi} - K_E)^*$, where $\tilde{\chi} = \{(e,\chi); \forall e \in E\}$ and $\tilde{\phi} = \{(e,\phi); \forall e \in E\}$ and soft point F_e^x is defined by $F_e^x(e) = \{x\}$ and $F_e^x(a) = \phi \, \forall e \neq a$ in E. For more information, we can refer to the sources mentioned above. Also the soft union $F_E \tilde{\cup} K_E = \{(e, F(e) \cup K(e));\ \forall e \in E)\}$, the soft intersection $F_E \tilde{\cap} K_E = \{(e, F(e) \cap K(e));\ \forall e \in E)\}$, the soft inclusion $F_E \tilde{\subseteq} K_E$ iff $F(e) \subseteq K(e);\ \forall e \in E$, and the soft complement $K_E^c = \{(e, \chi - K(e));\ \forall e \in E\}$. Now we give a new definition of different types of dense in soft ideal topological space via soft function.

2. Constructing Foundations

Definition 2.1: Let $(\tilde{\chi}, \tilde{\tau}, E, \tilde{J}_A)$ be a soft topological space over universally set χ and E the set of all possible parameters under consideration via to X, with $A \subseteq E$. Then the soft set F_A is called:

i. Soft someare locally dense (*Ds*-dense) iff $\Psi(F_A^*) \tilde{\neq} \tilde{\phi}$, we denoted the collection of all *Ds*-dense by $D_s(\tilde{\chi})$.
ii. Soft nowhere locally dense (*Dn*-dense) iff $\Psi(F_A^*) \tilde{=} \tilde{\phi}$, we denoted the collection of all *Dn*-dense by $D_n(\tilde{\chi})$.
iii. Soft complement someare locally dense (*Dc*-dense) iff F_A^c is *Ds*-dense, we denoted the collection of all *Dc*-dense by $D_C(\tilde{\chi})$.

In other words, using the properties of the soft local function and soft ψ-operator, the above definition in the following form, F_A is *Ds*-dense, if there exists $U_A \tilde{\in} \tilde{\tau}$ such that $U_A \tilde{\cap} \psi(F_A^c) \tilde{\in} \tilde{J}_A$, is called *Dc*-dense if there exists $U_A \tilde{\in} \tilde{\tau}$ such that $U_A \tilde{\cap} \psi(A) \tilde{\in} \tilde{J}_A$.

Example 2.2 : Let $\chi = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ and $\tilde{\tau} = \{\tilde{\tau}, \tilde{\phi}, F_{1E}, F_{2E}, F_{3E}, F_{4E}\}$ and $\tilde{J}_A = \{\tilde{\phi}, J_{1E}, J_{2E}, J_{3E}\}$ where $F_{1E} = \{(e_1, \{h_1\}), (e_2, \{h_1, h_3\})\}$, $F_{2E} = \{(e_1, \{h_2, h_3\}), (e_2, \{h_3\})\}$, $F_{3E} = \{(e_1, \phi), (e_2, \{h_3\})\}$, $F_{4E} = \{(e_1, \chi), (e_2, \{h_1, h_3\})\}$, $J_{1E} = \{(e_1, \phi), (e_2, \{h_2\})\}$, $J_{2A} = \{(e_1, \{h_1\}), (e_2, \phi)\}$, $J_{3E} = \{(e_1, \{h_1\}), (e_2, \{h_2\})\}$. Then if $H_E = \{(e_1, \{h_1, h_3\}), (e_2, \{h_2\})\}$, then $\psi(H_E^*) = \tilde{\phi}$. So H_E is D_n-dense and if $K_E = \{(e_1, \{h_1\}), (e_2, \{h_1, h_3\})\}$ then K_E is *Ds*-dense.

Definition 2.3 : Let $(\tilde{\chi}, \tilde{\tau}, E, \tilde{J}_E)$ be a soft ideal space. A soft set G_E is called *SL*-inner of the soft point F_e^*, if there exists a soft open set U_E such that $F_e^* \tilde{\in} U_E \tilde{\subseteq} G_E$. Noted that, if $\tilde{J}_E$ is a soft condense, then every *SL*-inner soft set of any soft point is *Ds*-dense. Also we easily to show that if F_E is *Ds*-dense and $F_E \tilde{\subseteq} H_E$, then H_E is also *Ds*-dense. There is also an important feature that links the two above definitions, which F_E is *Ds*-dense iff it is an *SL*-inner of at least one soft point in F_E.

Theorem 2.4:

1. *Every soft* $\tilde{J}_E - dense$ *is Ds-dense.*
2. *Every soft set* K_E *is either Ds-dense or Dc-dense.*

Proof : If possible that F_A is not *Ds*-dense, then $F_E^{*c^*} \cong \tilde{\chi}$, in other words that F_E^* is non-empty proper subset of $\tilde{\chi}$, but $F_E^{c^*} \cup F_E^* \cong \tilde{\chi}^*$. Now either $F_E^{c^*} \neq \tilde{\phi}$ or $F_E^{c^*} = \tilde{\phi}$, so, if $F_E^{c^*} \neq \tilde{\phi}$, then $\psi(F_E^{c^*}) \neq \tilde{\phi}$ iff F_E is *Dc*-dense, if $F_E^{c^*} \cong \tilde{\phi}$, then $F_E^* = \tilde{\chi}^*$, imply that $\psi(F_E) \cong \tilde{\chi}$, iff $\psi(F_E^{c^*}) = (\psi(F_E))^{*c} = \tilde{\chi}^{*c} \neq \tilde{\phi}$ iff F_E is *Dc*-dense.

We can easily to show that for any soft set K_E is either *Ds*-dense or *Dn*-dense in soft ideal space $(\tilde{\chi}, \tilde{\tau}, E, \tilde{J}_{F_e^\chi})$, where the soft ideal $\tilde{J}_{F_e^\chi} = \{H_E ; F_e^\chi \in H_E^c\}$.

The following theorem presents the important properties of D_s-dense.

Theorem 2.5 : *Let* $(\tilde{\chi}, \tilde{\tau}, E, \tilde{J}_E)$ *be soft ideal space, then the following are hold.*

i. If $\{K_{\lambda E}, \lambda \in \Lambda\}$ are collection of *Ds*-dense, then $\tilde{\cup}_{\lambda \in \Lambda} K_{\lambda E}$ is *Ds*-dense.
ii. For any collection $\{K_{\lambda E}, \lambda \in \Lambda\}$ of soft sets, if $\tilde{\cup}_{\lambda \in \Lambda} K_{\lambda E}$ is *Ds*-dense, then $K_{\lambda E}$ is *Ds*-dense $\forall \lambda \in \Lambda$.
iii. If $\forall \lambda \in \Lambda, K_{\lambda E}$ is *Dc*-dense then $\tilde{\cap}_{\lambda \in \Lambda} K_{\lambda E}$ is *Dc*-dense.
iv. If $\tilde{J}_E$ is soft condense and M_E is *Dc*-dense, then $\psi(M_E) \neq \phi$.
v. If $\tilde{J}_E$ is soft condense, then for each non-empty soft open set is *Ds*-dense.

Definition 2.6: The soft ideal space $(\tilde{\chi},\tilde{\tau},E,\tilde{J}_E)$ is called *SIS*-hyper connected, every soft *I*-dense F_E iff F_E is soft open set.

Proposition 2.7: Let the soft ideal space $(\tilde{\chi},\tilde{\tau},E,\tilde{J}_E)$ be *SIS*-hyper connected, then the following properties are satisfied.

i. If $\psi(K_E)=\psi(H_E)=\tilde{\phi}$, then $\psi(K_E\tilde{\cup}H_E)=\tilde{\phi}$.
ii. If K_E and H_E are *Dc*-dense, then $K_E\tilde{\cup}H_E$ is *Dc*-dense.
iii. If K_E and H_E are *Ds*-dense, then $K_E\tilde{\cap}H_E$ is *Ds*-dense.

The proof is directly from properties of the soft local function, soft ψ -operator and Definition 2.1 and Definition 2.3.

Theorem 2.8 : *For any soft set K_E in the soft ideal space $(\tilde{\chi},\tilde{\tau},E,\tilde{J}_E)$, we define the soft sets.*

(i) $SS(K_E)=\tilde{\cup}\{M_E\in D_s(\tilde{\chi});\ M_E\tilde{\subseteq}K_E\}$.
(ii) $SC(K_E)=\tilde{\cap}\{F_E\in D_c(\tilde{\chi});K_E\tilde{\subseteq}F_E\}$.
(iii) $SB(K_E)=SS(K_E)\cap SC(K_E{}^c)$.

Through the above definition we note that, if $\tilde{J}_E$ is soft condense $SB(K_E)$ is *Ds*-dense for K_E is soft closed set, also K_E is Ds-dense iff $SC(K_E)$ is *Ds*-dense.

Now by using the properties of Definition 2.1, we can prove the properties in the following proposition.

Proposition 2.9: For any soft sets K_E and H_E of soft ideal space $(\tilde{\chi},\tilde{\tau},E,\tilde{J}_E)$, the following features are truest.

(1) $K_E\tilde{\subseteq}SC(K_E)$ and $K_E\neq\tilde{\chi}$ is Dc - dense iff $K_E=SC(K_E)$.
(2) $SS(K_E)\subseteq K_E$ and $\tilde{\phi}\neq K_E$ is Ds - dense iff $K_E=SS(K_E)$.
(3) $[SS(K_E)]^c=SC(K_E^c)$.
(4) $[SC(K_E)]^c=SS(K_E^c)$.
(5) $SB(K_E)=[SS(K_E)\tilde{\cup}SS(K_E^c)]^c$.
(6) $SS(K_E)\tilde{\cup}SS(H_E)\tilde{\subseteq}SS(K_E\tilde{\cup}H_E)$.
(7) $SC(K_E)\tilde{\cup}SC(H_E)\tilde{\subseteq}SC(K_E\tilde{\cup}H_E)$.
(8) $SS(K_E\tilde{\cap}H_E)\tilde{\subseteq}SS(K_E)\cap SS(H_E)$.
(9) $SB(K_E)=SB(K_E^c)$.

(10) If $SC(K_E) = \tilde{\chi}$, then K_E^{*c} is soft I - dense.

(11) $SB(K_E)$ is Dc - dense.

(12) $\tilde{\phi} \neq K_E$ is Dc - dense iff $SB(K_E) \tilde{\cap} K_E = \tilde{\phi}$.

(13) $K_E \neq \tilde{\chi}$ is Dc - dense iff $SB(K_E) \subseteq K_E$.

(14) $\tilde{\phi} \neq K_E \tilde{\subset} \tilde{\chi}$ is both Ds - dense and Dc - dense iff $SB(K_E) = \tilde{\phi}$.

Proof of (13) : Let K_E is Ds-dense, then $K_E = SS(K_E)$ which imply that $\tilde{\phi} = SS(K_E) \tilde{\cap} SB(K_E) = K_E \tilde{\cap} SB(K_E)$. Conversely, since $SS(K_E) \subset K_E$, and for any soft point $F_e^x \tilde{\in} K_E$ which is not in $SB(K_E)$, so it is in $SS(K_E)$, hence $K_E = SS(K_E)$.

Proposition 2.10 : For any soft sets K_E and H_E in the soft ideal space $(\tilde{\chi}, \tilde{\tau}, E, \tilde{J}_E)$, it has the following features.

1. Either $SB(K_E) \subseteq K_E$ or $SB(K_E) \tilde{\subseteq} K_E^c$.
2. If $K_E \tilde{\subseteq} H_E$, then $SC(K_E) \subseteq SC(H_E)$ and $SS(K_E) \subseteq SS(H_E)$.
3. For any different soft points F_e^x, F_a^y, $(x \neq y \wedge e = a, \quad x = y \wedge e \neq a)$, $(x \neq y \wedge e \neq a)$ then $SC(F_e^x) \neq SC(F_a^y)$.

Definition 2.11 : If for each different soft points F_e^x, F_a^y, there exists D_s-dense sets G_E, H_E with $F_e^x \tilde{\in} G_E$ and $F_a^y \tilde{\notin} G_E$ and $F_a^y \in H_E$, $F_a^e \tilde{\notin} H_E$, the soft ideal space is called soft $ST1$-space.

Clearly that soft discrete topology space is soft $T1$-space and $ST1$-space for any soft ideal $\tilde{J}_E$. Also we noted that if the soft ideal $\tilde{J}_E$ is soft condense, then every soft $T1$-space is a soft $ST1$-space. Now let's introduce the main theorem.

Theorem 2.12 : *For any soft ideal space* $(\tilde{\chi}, \tilde{\tau}, E, \tilde{J}_E)$, *the following features are equivalent.*

i. $(\tilde{\chi}, \tilde{\tau}, E, \tilde{J}_E)$, *is soft ST1-space.*

ii. *For each soft point* F_e^x *is Dc-dense.*

iii. *For each soft set* $F_E = \tilde{\cap}\{K_E \in D_s(\tilde{X}); F_E \tilde{\subseteq} K_E\}$.

3. Discussion and Conclusion

We can define the others separation axioms as well as the definition of soft $ST1$-space and we study the relationship between them, also study the important features of these separation axioms, we can develop the soft local function by using soft ω-open and define the Ds-dense and *Dc*-dense via this development. The results in the papers [9,10,11] can be modified by using the *Ds*-dense.

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