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ORIGINAL ARTICLE

An elliptic equation of finite index in a domain

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Abstract
We give an example of first order elliptic equation for a complex-valued function in a plane domain which has a finite number of linearly independent solutions for any right-hand side. No boundary value conditions are thus required.

Keywords Elliptic equation · Fredholm operator · Index

Mathematics Subject Classification Primary 35F20 · Secondary 35J60 · 35J46

1 The main result

Let $\mathcal{X}$ be a simply connected bounded domain in the complex plane $\mathbb{C}$. The boundary of $\mathcal{X}$ is assumed to be of Lyapunov class $C^{1,\lambda}$, where $0<\lambda\leq 1$. Consider the equation

$$\partial_{\bar{z}}(u + a(z)\bar{u}) = f \tag{1.1}$$

for an unknown complex-valued function u in $\mathcal{X}$, where a and f are given functions in the domain.

Obviously, Eq. (1.1) fails to be linear over the field of complex numbers, for it contains the conjugate of u. It reduces to a system of two first order partial differential equations with real coefficients for two real-valued functions $u^1 = \Re u$

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