On pair wise α-continuous and pair wise pre continuous mappings

zahir dobeas al-nfie
department of math.college of education , Babylon university

abstract
[A.S.mashhour , I.A.hasanein and S.N.el-deeb] in 1983 studied sevsral of α-continuous and α-open mapping in topological spaces in this search we show that results similar th these in bitopolopological spaces.

الخلاصة:
هذا البحث يتناول دراسة موضوع دوال ألفا المستمرة ودوال الفا المفتوحة المفتوحة في الفضاءات ثنائية لاالتبولوجي والتي هي عبارة عن مجموعة معرف عليها فضائيين تبولوجين في ان واحد كذلك دراسة العلاقة بين هذه الدوال ودوال معرفة سابقا

Introduction

Let X,Y,Z be topological spaces on which no separation axioms are assumed unless explicitly stated , let S be a subset of X , the closure (resp. interior) of S will be denoted by cl(s) (resp. int(S)) . a subset of S of X is called α-set [5] (resp. semi-open set[3] , pre open set [4]) if $S\subset int(cl(int(S))$ (resp. $S\subset cl(int(S))$, $S\subset int(cl(S))$) , the complement of an α-set (resp. semi-open set, preopen set) is called α-closed (resp. semi-closed , pre closed) the space of all α-set(semi-open ,pre open) is denoted α(X)(resp. SO(X),PO(X)) .it is clear that each α-set is semi-open and pre open and the converse is not true. A mapping $f:X\rightarrow Y$ is called almost continuous [7] if for each $x\in X$ and each open neighborhood V of f(x) there exist an open neighborhood U of x such that $f(U)\subset int(cl(V))$, and it is called θ-continuous if $f(U)\subset cl(V)$, a mapping $f:X\rightarrow Y$ is called seif the inverse image of each open set in Y is mi-open set in X and f is called α-continuous if the inverse image of each open set is an α-open set in X [2], and if it is called α-open if the image of each open set is α-open set Y.

2- Pair wise α-continuity

Definition 2-1: a bitopological space X is anon empty set X with two topologies t1,t2 defined on it , in other ward is the triple (X,t_1,t_2) such that (X,t1) and (X,t2) are two topologies on the same set X.

Definition 2-2: let (X,t_1,t_2) and (Y,p1,p2) are two bitopological spaces a mapping $f:X\rightarrow Y$ is said to be pair wise α-continuous iff the induced maps $g:(X,t_1)\rightarrow(Y,p_1)$ and $h:(X,t_2)\rightarrow(Y,p_2)$ are α-continuous mapping .

Definition 2-3: a mapping $f: (X,t_1,t_2)\rightarrow (Y,p_1,p_2)$ is said to be pair wise θ-continuous iff the induced maps $g:(X,t_1)\rightarrow(Y,p_1)$ and $h:(X,t_2)\rightarrow(Y,p_2)$) are θ-continuous .

Definition2-4: a mapping $f: (X,t_1,t_2)\rightarrow (Y,p_1,p_2)$ is said to be pairwise semi-continuous iff the induced maps $g:(X,t_1)\rightarrow(Y,p_1)$ and $h:(X,t_2)\rightarrow(Y,p_2)$ are semi-continuous .

Definition2-5: a mapping $f: (X,t_1,t_2)\rightarrow (Y,p_1,p_2)$is said to be pair wise pre-continuous iff the induced maps $g:(X,t_1)\rightarrow(Y,p_1)$ and $h:(X,t_2)\rightarrow(Y,p_2)$ are pre-continuous .

Definition2-6: a mapping $f: (X,t_1,t_2)\rightarrow (Y,p_1,p_2)$ is said to be pair wise almost-continuous iff the induced maps $g:(X,t_1)\rightarrow(Y,p_1)$ and $h:(X,t_2)\rightarrow(Y,p_2)$ are almost-continuous .

Theorem2-1: let f: $(X,t_1,t_2)\rightarrow(Y,p_1,p_2)$ be a mapping , then the following statement are equivalent:

i- f is pair wise α-continuous

ii- each $x\in X$ and each open set $V\subset Y$ containing f(x) there exist $W\subset X$ such that $x\in W$,$f(W)\subset V$.

iii- the inverse image of each closed set in Y is α-closed set.

Proof: $(i\rightarrow ii)$

since f is pair wise α-continuous , then the induced maps $g:(X,t_1)\rightarrow(Y,p_1)$ and $h:(X,t_2)\rightarrow(Y,p_2)$ are α-continuous mapping and by definition of α-continuity in topological space the result exist.

Proof$(ii\rightarrow iii)$

if we let (ii) then we have that $f:X\rightarrow Y$ is an α-continuous and by lating the complement of the open set and the complement of α-open set we have (iii).

Corollary 2-1:

Let f: $(X,t_1,t_2)\rightarrow(Y,p_1,p_2)$ be pair wise α-continuous then

i- $f(cl(A))\subset cl(f(A))$ for each $A\in PO(X)$

ii- ii- $cl(f^{-1}(M))\subset f^{-1}(cl(M))$ for each $M\in PO(X)$

.

Proof: see [1] and [6]

Theorem 2-2: every pair wise α-continuous mapping is pair wise θ-continuous.

Proof: let f: $(X,t_1,t_2)\rightarrow(Y,p1,p2)$ be pair wise α-continuous this implies that the induced maps $g:(X,t_1)\rightarrow(Y,p_1)$ and $h:(X,t_2)\rightarrow(Y,p_2)$ are α-continuous

Now to prove that f is pair wise θ-continuous we must prove that the induced maps g and f are θ-continuous

Now we have $g:(X,t_1)\rightarrow(Y,p_1)$ is α-continuous . let $x \in X$ and $V\subset Y$ be an open set containing g(x) , by theorem (1-1) in [ ] $cl(int(cl(g^{-1}(V)))\subset g^{-1}(cl(V))$, since g is α-continuous we have $g^{-1}(V)\subset int(cl(int(g^{-1}(V)))\subset cl(g^{-1}(int(cl(int(V)))\subset cl(int(cl(g^{-1}(V)))\subset g^{-1}(cl(V))$, put $int(cl(int(g^{-1}(V))=U$, so U is neighborhood of x such that $cl(U)\subset g^{-1}(clV))$, namely $g(cl(U)))\subset V$ there for g is θ-continuous , similarly we can prove that h is θ-continuous Which is mean that f is pair wise θ-continuous.

Remark(2-1): it is clear that the class of pair wise α-continuity contains the class of pair wise continuity but it is contained in the class of pair wise θ-continuity . pair wise pre-continuity and the concepts of pair wise α-continuity and pair wise almost-continuity independent. the following diagram summarized the above discussion:

pair wise semi-continuous

⇑

pair wise continuity ⇒⇒⇒pair wise α-continuity⇒pair wise pre-continuity

⇓ ⇓

pair wise almost-continuity ⇒ pair wise θ-continuity

The example given below show that the converse of these implication are not true in general.

Example 2-1: a mapping f: $(X,t_1,t_2) \rightarrow (Y,p1,p2)$ such that if (X,t1) and (X,t2) are indiscrete spaces and (Y,p1),(Y,p2) are discrete spaces then g:(X,t1)$\rightarrow$(Y,p1) is pre continuous but it is not α-continuous , similarly h:$(X,t_2)\rightarrow(Y,p_2)$, which is mean that f is pair wise pre continuous but it is not pair wise α-continuous .

Example2-2: let Y=X={1,2,3}, t1={X,Ø, {1},{3},{1,3}}, t2={X,Ø,{2},{3{2,3}} such that g(1)=1,g(2)=2=g(3) and h(1)=h(2)=2,h(3)=3 and p1,p2 are discrete spaces then g:(X,t1)$\rightarrow$(Y,p1) and h:(X,t2)$\rightarrow$(Y,p2) are semi-continuous but not α-continuous which is mean that f: $(X,t_1,t_2) \rightarrow$ (Y,p1,p2) is pair wise semi-continuous but not pair wise α-continuous.

Example2-3: let Y=X={a,b,c}, t1={X,Ø, {a},}, t2={X,Ø,{b},}, p1={Y, {a}}, p2={Y,{b}} such that g:(X,t1)$\rightarrow$(Y,p1) is defined by g(a)=g(b)=a,g(c)=c and h:(X,t2)$\rightarrow$(Y,p2) is defined by h(a)=h(b)=b,h(c)=c , then g,h are α-continuous but not continuous which is mean that f: $(X,t_1,t_2) \rightarrow$ (Y,p1,p2) is pair wise α-continuous but not pair wise continuous.

References:

1- T. husain,: almost contiuouse mappings: prace Math.loc(1966), 1-7.
2- N.Levine, :Adecomposition of contiuity in topological spaces,Ameer-math,Monthly (1961), 44-46.

3- N.Levine, : semi-open sets and semi-continuity in topogical spaces" Ameer,Math, monthly 70(1963), 36-41.
4- A.S.Mashhour, M.E.Abd-monsef and S.N. WEl-deeb,"on pre-continuous and week continuous mappings" procphys,SOC,Egypt,51(1981).
5- A.S.Mashhour, I.A.Hasnein and S.N.El-deeb," α-ontinuous and α-open mappings" Acta Math.Hung,41(1983), 213-218
6- T.noiri ,"Ageneralization of closed mappings " , Atti.Accad, Naz.Lincei.rend,Cl.SCI,nsatur 54(1973), 412-415.
7- M.K.Singal and A.R.Sngal, "almost continous functions ,Yokohama, Mth.J.16(1968),63-73.