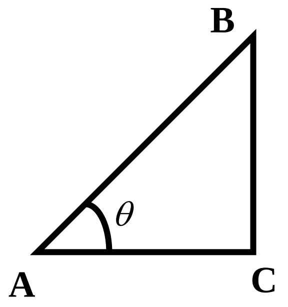
Basic Concept



$1-\quad \sin(\theta)=\dfrac{BC}{AB}$ $\qquad 2-\quad \cos(\theta)=\dfrac{AC}{AB}$

$$3-\quad \tan(\theta)=\frac{\sin(\theta)}{\cos(\theta)}=\frac{\frac{BC}{AB}}{\frac{AC}{AB}}=\frac{BC}{AC}$$

$4-\quad \cot(\theta)=\dfrac{1}{\tan(\theta)}=\dfrac{1}{\frac{BC}{AC}}=\dfrac{AC}{BC}$ $\qquad 5-\quad \sec(\theta)=\dfrac{1}{\cos(\theta)}=\dfrac{AB}{AC}$ $\qquad 6-\quad \csc(\theta)=\dfrac{1}{\sin(\theta)}=\dfrac{AB}{BC}$

Relations

$$\sin^2(x)+\cos^2(x)=1$$

$\div\cos^2(x)$:

$$\frac{\sin^2(x)}{\cos^2(x)}+\frac{\cos^2(x)}{\cos^2(x)}=\frac{1}{\cos^2(x)}$$

$$\Downarrow$$

$$\tan^2(x)+1=\sec^2(x)$$

$\div\sin^2(x)$:

$$\frac{\sin^2(x)}{\sin^2(x)}+\frac{\cos^2(x)}{\sin^2(x)}=\frac{1}{\sin^2(x)}$$

$$\Downarrow$$

$$1+\cot^2(x)=\csc^2(x)$$

$$\sin^2(x)=\frac{1-\cos(2x)}{2} \qquad \cos^2(x)=\frac{1+\cos(2x)}{2}$$

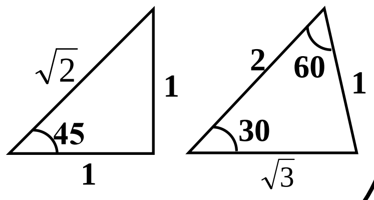
$$\sin(2x)=2\sin(x)\cos(x) \qquad \cos(2x)=\cos^2(x)-\sin^2(x)$$

$$\sin(-x)=-\sin(x) \qquad \cos(-x)=\cos(x)$$

$$\sin(a-\frac{\pi}{2})=-\cos(a) \qquad \cos(a-\frac{\pi}{2})=\sin(a)$$

Special angles

θ	θ	$\sin(\theta)$	$\cos(\theta)$
0	0	**0**	**1**
$\pi/2$	90	**1**	**0**
π	180	**0**	**-1**
$3\pi/2$	270	**-1**	**0**
2π	360	**0**	**1**



$$\sin(a\pm b)=\sin(a)\cos(b)\pm\cos(a)\sin(b)$$

$$\cos(a\pm b)=\cos(a)\cos(b)\mp\sin(a)\sin(b)$$

$$\tan(a+b)=\frac{\tan(a)+\tan(b)}{1-\tan(a)\tan(b)} \qquad \tan(a-b)=\frac{\tan(a)-\tan(b)}{1+\tan(a)\tan(b)}$$

E-mail aljelawy2000@yahoo.com

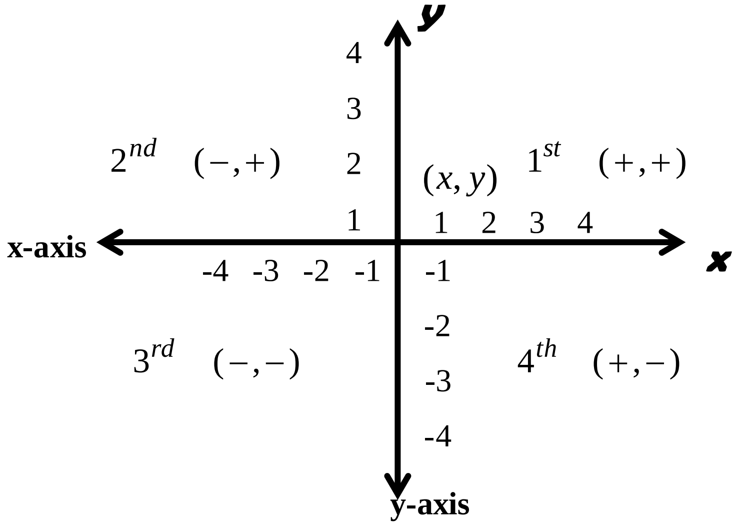
Coordinates For the plane

Sketch

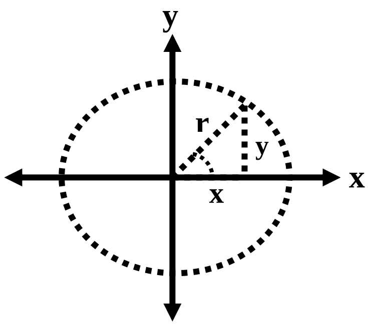
1- (1 ,2)

2- (0 , 1)

3- (-1 , -3)



Show That $\sin^2(\theta)+\cos^2(\theta)=1$



Prove :- we have $\sin(\theta)=\frac{y}{r} \Rightarrow y=r\sin(\theta)$

$$\cos(\theta)=\frac{x}{r} \Rightarrow x=r\cos(\theta)$$

$x^2+y^2=r^2$ Equation of Circle

$$[r\cos(\theta)]^2+[r\sin(\theta)]^2=r^2$$

$$r^2\cos^2(\theta)+r^2\sin^2(\theta)=r^2 \Rightarrow \cancel{r^2}\left(\cos^2(\theta)+\sin^2(\theta)\right)=\cancel{r^2}$$

$$\therefore \cos^2(\theta)+\sin^2(\theta)=1$$

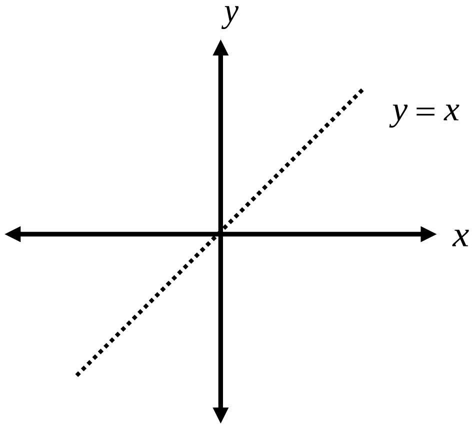
E-mail aljelawy2000@yahoo.com

Graph The Function

Sketch $y = x$ $y = x^2$ $3y + x = 4$ ***(H.w)***

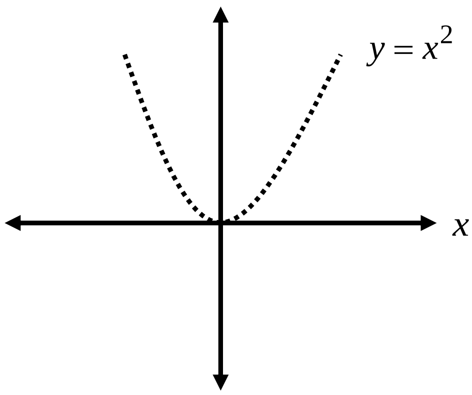
1- $y = x$

x	y	(x,y)
1	1	(1,1)
2	2	(2,2)
0	0	(0,0)
-1	-1	(-1,-1)
-2	-2	(-2,-2)



2- $y = x^2$

x	y	(x,y)
1	1	(1,1)
2	4	(2,4)
0	0	(0,0)
-1	1	(-1,1)
-2	4	(-2,4)



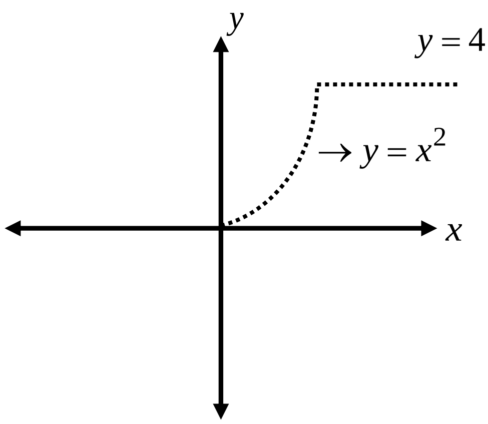
Graph $y = \begin{cases} x^2 & 0 \le x \le 2 \\ 4 & x \ge 2 \end{cases}$

$y = x^2$

x	y	(x,y)
0	0	(0,0)
1	1	(1,1)
2	4	(2,4)

$y = 4$

x	y	(x,y)
2	4	(2,4)
3	4	(3,4)
4	4	(4,4)
5	4	(5,4)
6	4	(6,4)



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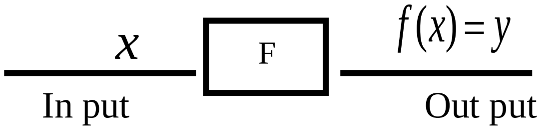
The function

Is the rule that assign each value of independent variable to single value of dependent variable



$f(x) = y$

x :− **Independent variable**

$f(x) = y$:− **dependent**

If The Function $y(x) = x^2 + 1$, $y = \sqrt{x+2}$

Find $y(1)$, $y(x+7)$, $y(0)$, $y(a+b)$

Derivatives

The derivative of the function $y = f(x)$ **is the function** $y' = f'(x)$ **Whose value at each** x **is define by rule** $y = f(x) \quad \Rightarrow \dfrac{dy}{dx} = \lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x)}{\Delta x} = y' = f'(x)$

🛠 The Rules for Derivative

1	**If** $y = b \Rightarrow \dfrac{dy}{dx} = o$ **where** b **is constant**	$y = a^4 \Rightarrow \dfrac{dy}{dx} = 0$
2	**If** $y = x^n \Rightarrow \dfrac{dy}{dx} = nx^{n-1}$ n **any number**	$y = x^{-2} \Rightarrow \dfrac{dy}{dx} = -2x^{-2-1} = -2x^{-3}$
3	**If** $y = bx^n \Rightarrow \dfrac{dy}{dx} = b.nx^{n-1}$	$y = 4.\sqrt[3]{x} \Rightarrow \dfrac{dy}{dx} = 4.\dfrac{1}{3}x^{\frac{1}{3}-1} = \dfrac{4}{3.\sqrt[3]{x^2}}$
4	**If** $y = u(x) + v(x) \Rightarrow \dfrac{dy}{dx} = \dfrac{du}{dx} + \dfrac{dv}{dx}$	$y = 2x^2 + 8 - 5x^4 \Rightarrow \dfrac{dy}{dx} = 4x + 0 - 20x^3$
5	**If** $y = b[u(x)]^n \Rightarrow \dfrac{dy}{dx} = b.n[u(x)]^{n-1}.\dfrac{du}{dx}$	$y = 3(2x^2 - x + 4)^7 \Rightarrow \dfrac{dy}{dx} = 3.7(2x^2 - x + 4)^6.(4x-1)$
6	**If** $y = u(x).v(x) \Rightarrow \dfrac{dy}{dx} = u(x).\dfrac{dv}{dx} + v(x).\dfrac{du}{dx}$	$y = (x^2+1)(x-3)^2$ $\Rightarrow \dfrac{dy}{dx} = (x^2+1)[2(x-3)] + (x-3)^2(2x)$
7	**if** $y = \dfrac{u(x)}{v(x)} \Rightarrow \dfrac{dy}{dx} = \dfrac{v(x).\dfrac{du}{dx} - u(x).\dfrac{dv}{dx}}{[v(x)]^2}$	$y = \dfrac{(x^2+1)^2}{(3x^2-2x+6)^2}$ 

The derivative of composite functions (Chain rule)

If y **is differentiable function of (**u**) and (**u**) is differentiable function of (** x**) Then** y **is a differentiable function of (** x**) That is**

$$y = f(u) \Rightarrow \frac{dy}{du} \qquad u = f(x) \Rightarrow \frac{du}{dx} \qquad \Rightarrow \quad \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$$

E-mail aljelawy2000@yahoo.com ✍

Find $\frac{dy}{dt}$ **where** $y = x^2 + \sqrt{x}$ **and** $x = 3t^2 - 2t + 1$

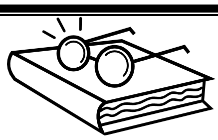
Solution:- $\frac{dy}{dt} = \frac{dy}{dx}.\frac{dx}{dt} \Rightarrow \frac{dy}{dt} = (2x + \frac{1}{2\sqrt{x}}).(6t - 2)$

Substitute: $x = 3t^2 - 2t + 6$

$$\frac{dy}{dx} = [2(3t^2 - 2t + 6) + \frac{1}{2\sqrt{3t^2 - 2t + 6}}](6t - 2)$$

Find $\frac{dy}{dx}$ **where** $x = 2t + 3$ **and** $y = t^2 - 1$

Solution:- $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t}{2} = t = \frac{x-2}{2}$



How To Solve

1 **Find** $\frac{dy}{dt}$ **by the chain rule expressing the results in terms of** t

$y = x^2 + \frac{x}{2}$, $x = 2t - 5$ $\qquad y = \sqrt{x+2}$, $x = \frac{2}{t}$ $\qquad y = \frac{x^2}{x^3 + 1}$, $x = \sqrt{2t^2 + t + 1}$

2 **Find** $\frac{dz}{dx}$ **if** $z = w^2 - w^{-1}$, $w = 3x$ **Find** $\frac{da}{db}$ **if** $a = 7r^3 - 2$, $r = 1 - \frac{1}{b}$

3 **Find** $\frac{dr}{dt}$ **if** $r = (s+1)^{\frac{1}{2}}$, $s = 10t^2 - 2t$ **Find** $\frac{dy}{dt}$ **if** $x = 3t + 1$, $y = t^{-3} + \sqrt{t}$

E-mail aljelawy2000@yahoo.com

Implicit derivative

 **Find** $\frac{dy}{dx}$ **When** $y^3 + xy + x^2 = 2$

$$3y^2 \frac{dy}{dx} + x\frac{dy}{dx} + y + 2x = 0$$

$$(3y^2 + x).\frac{dy}{dx} = -\frac{(y+2x)}{3y^2 + x}$$

E-mail aljelawy2000@yahoo.com

Hyperbolic Function

$$\sinh(x)=\frac{e^{x}-e^{-x}}{2} \qquad \cosh(x)=\frac{e^{x}+e^{-x}}{2}$$

$$\tanh(x)=\frac{e^{2x}-1}{e^{2x}+1} \qquad \coth(x)=\frac{e^{2x}+1}{e^{2x}-1}$$

$$\operatorname{sec}h(x)=\frac{1}{\cosh(x)}=\frac{2}{e^{x}+e^{-x}} \qquad \csc(x)=\frac{1}{\sinh(x)}=\frac{2}{e^{x}-e^{-x}}$$

Derivative

Trigonometric Functions	Hyperbolic Trigonometric Function
$y=\sin(u) \Rightarrow \frac{dy}{dx}=\cos(u).u'$	$y=\sinh(u) \Rightarrow \frac{dy}{dx}=\cosh(u).u'$
$y=\cos(u) \Rightarrow \frac{dy}{dx}=-\sin(u).u'$	$y=\cosh(u) \Rightarrow \frac{dy}{dx}=\sinh(u).u'$
$y=\tan(u) \Rightarrow \frac{dy}{dx}=\sec^{2}(u).u'$	$y=\tanh(u) \Rightarrow \frac{dy}{dx}=\operatorname{sec}h^{2}(u).u'$
$y=\cot(u) \Rightarrow \frac{dy}{dx}=-\csc^{2}(u).u'$	$y=\coth(u) \Rightarrow \frac{dy}{dx}=-\operatorname{csc}h^{2}(u).u'$
$y=\sec(u) \Rightarrow \frac{dy}{dx}=\sec(u)\tan(u).u'$	$y=\operatorname{sec}h(u) \Rightarrow \frac{dy}{dx}=-\operatorname{sec}h(u)\tanh(u).u'$
$y=\csc(u) \Rightarrow \frac{dy}{dx}=-\csc(u)\cot(u).u'$	$y=\operatorname{csc}h(u) \Rightarrow \frac{dy}{dx}=-\operatorname{csc}h(u)\coth(u).u'$

1- $y=\sin^{2}(x^{2})\Rightarrow\frac{dy}{dx}=2\sin(x^{2}).\cos(x^{2}).(2x)$

2- $y=\sec^{2}(3x+1)\Rightarrow\frac{dy}{dx}=2\sec(3x+1).\sec(3x+1)\ \ \tan(3x+1)\ \ .3$

3- $y=\frac{\tan(x)}{\sec x)}\Rightarrow\frac{dy}{dx}=\frac{\sec(x).\sec^{2}(x)-\tan(x).\sec(x).\tan(x)}{\sec^{2}(x)}$

4- $y=\tan(3x)\Rightarrow\frac{dy}{dx}=\sec^{2}(3x).3\ \ =3\sec^{2}(3x)$

E-mail aljelawy2000@yahoo.com

Derivative

Natural Logarithms $\ln(x)$	**Exponential Function** e^x
If $u(x)$ **is differential function of** (x) **and** $y=\ln[u(x)] \Rightarrow \frac{dy}{dx}=\frac{du/dx}{u(x)}$	**If** $u(x)$ **is differential function of** (x) **and** $y=e^{u(x)} \Rightarrow \frac{dy}{dx}=e^{u(x)}.\frac{du}{dx}$
1- $y=\ln(x^2) \Rightarrow \frac{dy}{dx}=\frac{2x}{x^2}$ 2- $y=\ln(x^2+3x-7) \Rightarrow \frac{dy}{dx}=\frac{2x+3}{x^2+3x-7}$ 3- $y=\ln[\sin^2(x)] \Rightarrow \frac{dy}{dx}=\frac{2\sin(x)\cos(x)}{\sin^2(x)}$	**1-** $y=e^{x^2} \Rightarrow \frac{dy}{dx}=e^{x^2}(2x)$ **2-** $y=e^{x\sin(x)} \Rightarrow \frac{dy}{dx}=e^{x\sin(x)}[x\cos(x)+\sin(x)]$ **3-** $y=e^{\tan(x)} \Rightarrow \frac{dy}{dx}=e^{\tan(x)}[\sec^2(x)]$

Properties Of Natural Logarithms		**Properties Of Exponential Function**	
$\ln(x.y)=\ln(x)+\ln(y)$	$\ln(\frac{x}{y})=\ln(x)-\ln(y)$	$e^{x+y}=e^x.e^y$	$e^{x-y}=\frac{e^x}{e^y}$
$\ln(x^r)=r\ln(x)$	$\ln(1)=0$	$e^{\ln(x)}=x$	$(e^x)^r=e^{rx}$

Derivative of Inverse Of trigonometric function

If $u(x)$ **is differential function of** x $\quad y=\sin^{-1}(x) \overset{Iff}{\Leftrightarrow} x=\sin(y)$

$y=\sin^{-1}(u) \Rightarrow \frac{dy}{dx}=\frac{u'}{\sqrt{1-(u)^2}}$	$y=\cos^{-1}(u) \Rightarrow \frac{dy}{dx}=\frac{-u'}{\sqrt{1-(u)^2}}$
$y=\tan^{-1}(u) \Rightarrow \frac{dy}{dx}=\frac{u'}{1+(u)^2}$	$y=\cot^{-1}(u) \Rightarrow \frac{dy}{dx}=\frac{-u'}{1+(u)^2}$
$y=\sec^{-1}(u) \Rightarrow \frac{dy}{dx}=\frac{u'}{u\sqrt{(u)^2-1}}$	$y=\csc^{-1}(u) \Rightarrow \frac{dy}{dx}=\frac{-u'}{u\sqrt{(u)^2-1}}$

E-mail aljelawy2000@yahoo.com

Some Important Properties the inverse of trigonometric function

1	$\sin^{-1}(-x)=-\sin^{-1}(x)$	$\cos^{-1}(-x)=\pi-\cos^{-1}(x)$	$\tan^{-1}(-x)=-\tan^{-1}(x)$
2	$\cot^{-1}(-x)=-\cot^{-1}(x)$	$\sec^{-1}(-x)=\pi-\sec^{-1}(x)$	$\csc^{-1}(-x)=-\csc^{-1}(x)$
3	$\sin^{-1}(-x)=\frac{\pi}{2}-\cos^{-1}(x)$	$\tan^{-1}(-x)=\frac{\pi}{2}-\cot^{-1}(x)$	$\sec^{-1}(-x)=\frac{\pi}{2}-\csc^{-1}(x)$
4	$\sin^{-1}(x)=\csc^{-1}(\frac{1}{x})$	$\cos^{-1}(x)=\sec^{-1}(\frac{1}{x})$	$\tan^{-1}(x)=\cot^{-1}(\frac{1}{x})$
5	$\sin^{-1}[\sin(x)]=x$	$\sin[\sin^{-1}(x)]=x$	

Some Important Properties the inverse of Hyperbolic function

1	$\cosh^2(x)-\sinh^2(x)=1$	$\tanh^2(x)+\sec h^2(x)=1$	$\coth^2(x)-\csc h^2(x)=1$
2	$\cosh(-x)=\cosh(x)$	$\sinh(-x)=-\sinh(x)$	$\tanh(-x)=-\tanh(x)$
3	$\sinh(x\pm y)=\sinh(x)\cosh(y)\pm\cosh(x)\sinh(y)$		$\cosh(x)\pm\sinh(x)=e^{\mp x}$
4	$\cosh(x\pm y)=\cosh(x)\cosh(y)\mp\sinh(x)\sinh(y)$		$\sinh(2x)=2\sinh(x)\cosh(x)$

Show that $\cosh^2(x)-\sinh^2(x)=1$

$$\cosh^2(x)-\sinh^2(x)=\left(\frac{e^x+e^{-x}}{2}\right)^2-\left(\frac{e^x-e^{-x}}{2}\right)^2=\frac{e^{2x}+2+e^{-2x}}{4}-\frac{e^{2x}-2+e^{-2x}}{4}=\frac{4}{4}=1$$

Show that **1-** $\sin^{-1}(-x)=-\sin^{-1}(x)$ **2-** $\sin^{-1}(-x)=\frac{\pi}{2}-\cos^{-1}(x)$

1- Let $y=\sin^{-1}(-x) \Rightarrow -x=\sin(y) \Rightarrow x=-\sin(y) \Rightarrow x=\sin(-y) \Rightarrow -y=\sin^{-1}(x) \Rightarrow y=-\sin^{-1}(x)$

2- Let $y=\sin^{-1}(-x) \Rightarrow -x=\sin(y) \Rightarrow x=\cos(\frac{\pi}{2}-y) \Rightarrow \frac{\pi}{2}-y=\cos^{-1}(x) \Rightarrow y=\frac{\pi}{2}-\cos^{-1}(x)$

$y=\frac{\pi}{2}-\cos^{-1}(x)$

If $x=\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ **Find** $\cos(x)$, $\tan(x)$

Let $x=\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \Rightarrow \frac{\sqrt{3}}{2}=\frac{\pi}{3} \Rightarrow \sin^{-1}(\frac{\sqrt{3}}{2})=\frac{\pi}{3}$

$$\cos^2(x)+\sin^2(x)=1 \Rightarrow \cos(x)=\sqrt{1-\sin^2(x)}=\sqrt{1-\frac{3}{4}}=\sqrt{\frac{1}{4}}=\frac{1}{2}$$

E-mail aljelawy2000@yahoo.com

Solve For x **If** $\tan^{-1}(x)-\cot^{-1}(x)=\dfrac{\pi}{4}$

$$\Rightarrow \tan^{-1}-\left(\frac{\pi}{2}-\tan^{-1}(x)\right)=\frac{\pi}{4} \;\Rightarrow\; 2\tan^{-1}(x)=\frac{\pi}{4}+\frac{\pi}{2}=\frac{3\pi}{4} \;\Rightarrow \tan^{-1}(x)=\frac{3\pi}{8} \Rightarrow x=\tan\left(\frac{3\pi}{8}\right)$$

If $y=\sin^{-1}(x)$ **Prove that** $\dfrac{dy}{dx}=\dfrac{1}{\sqrt{1-x^2}}$

Solution:-

$$y=\sin^{-1}(x) \;\Rightarrow\; x=\sin(y) \;\Rightarrow\; 1=\cos(y)\frac{dy}{dx} \;\Rightarrow\; \frac{dy}{dx}=\frac{1}{\cos(y)}=\frac{1}{\sqrt{1-\sin^2(y)}}=\frac{1}{\sqrt{1-x^2}}$$

Find $\dfrac{dy}{dx}$

1- IF $y=\sin^{-1}(2x^2) \Rightarrow \dfrac{dy}{dx}=\dfrac{4x}{\sqrt{1-(2x^2)^2}}$ **2- If** $y=\tan^{-1}(x^2+2x\ ) \Rightarrow \dfrac{dy}{dx}=\dfrac{2x+2}{1+(x^2+2x)^2}$

2- IF $y=\sin^{-1}[x^2+3x-\cos(x)] \Rightarrow \dfrac{dy}{dx}=\dfrac{2x-3-[-\sin(x)]}{\sqrt{1-\left(x^2+3x-\cos(x)\right)^2}}$

3- IF $y=\cos^{-1}[x^2+\tan^2(2x)] \Rightarrow \dfrac{dy}{dx}=-\dfrac{(2x+2\tan(2x)\sec^2(2x)2}{\sqrt{1-[x^2+\tan^2(2x)]^2}}$

Find $\dfrac{dy}{dx}$

1- If $y=[\sin(x)]^x \;\Rightarrow\; \ln(y)=\ln[\sin(x)]^x \;\Rightarrow\; \ln(y)=x\ln[\sin(x)]$

$$\frac{1}{y}.\frac{dy}{dx}=x.\frac{\cos(x)}{\sin(x)}+\ln[\sin(x)] \;\Rightarrow\; \frac{dy}{dx}=y\left(x.\frac{\cos(x)}{\sin(x)}+\ln[\sin(x)]\right)$$

$$\frac{dy}{dx}=[\sin(x)]^x\left(x.\frac{\cos(x)}{\sin(x)}+\ln[\sin(x)]\right)$$

2- IF $y=[x+1]^2.e^{x+1}.\tan(x^2).\csc^{-1}(2x+1) \Rightarrow \ln(y)=\ln\left([x+1]^2.e^{x+1}.\tan(x^2).\csc^{-1}(2x+1)\right)$

$$\frac{y'}{y}=\frac{2}{x+1}+1+\frac{\sec^2(x^2)2x}{\tan(x^2)}+\frac{\dfrac{-2}{(2x+1)\sqrt{(2x+1)^2-1}}}{\csc^{-1}(2x+1)}$$

E-mail aljelawy2000@yahoo.com

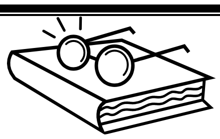
3- IF $y = \sin^2[\sec^{-1}(2x)]\cot^{-1}(x)$

$$\frac{dy}{dx} = \sin^2(\sec^{-1}(2x)].\frac{-1}{1+x^2} + \cot^{-1}(x).2\sin(\sec^{-1}(2x)).\cos(\sec^{-1}(2x).\frac{2}{2x\sqrt{(2x)^2-1}}$$

Find $\frac{dy}{dx}$, $y = \tan^{-\frac{1}{2}}[\sec^3(x^2+\sin(2x)]$

$$\frac{dy}{dx} = -\frac{1}{2}\tan^{-\frac{3}{2}}[\sec^3(x^2+\sin(2x)]\sec^2[\sec^3(x^2+\sin(2x))]$$

$$3\sec^2(x^2+\sin(2x))[\sec(x^2+\sin(2x)]\tan[\sec(x^2+\sin(2x)][2x+2\cos(2x)]$$

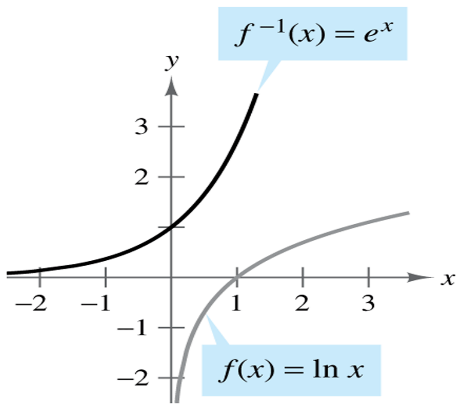


How To Solve

Find $\frac{dy}{dx}$

1	$y = \frac{\cos(x)}{1-\sin(x)}$	$y = \sec^4(x^2+1)$	$y = \sin(x^3-2x+7)$	$x + \tan(xy) = 0$
2	$y^2 - 6\sin(x) + 4y = 0$	$y = \tan(3x).\sec(4x)$	$y = \ln[\sin^2(2x+1)^2]$	$y = e^{\tan(x)}.\sin^{-1}(x)$
3	$\lvert x-2\rvert \le 4$	$\left\lvert\frac{x-2}{6}\right\rvert < 1$	$\left\lvert\frac{2}{x-1}\right\rvert \ge 1$	$\left\lvert\frac{1}{x}\right\rvert \le 2$

Indeterminate forms	
$\frac{0}{0}$	Meaning Less
$\frac{\infty}{\infty}$	Meaning Less
$0 * \infty$	Meaning Less
1^{∞}	Meaning Less
∞^0	Meaning Less
0^0	Meaning Less
$\infty - \infty$	Meaning Less



$$\log_b(x) = \frac{\ln(x)}{\ln(b)}$$

Determinate forms
$\frac{0}{\infty} \to 0$
$\frac{\infty}{0} \to 0$
$\infty.\infty \to \infty$
$0^{-\infty} \to \infty$
$0^{\infty} \to 0$
$\infty + \infty \to \infty$
$1^0 \to 1$

E-mail *aljelawy2000@yahoo.com*

The Limits

Find The Following Limits

1- $\lim\limits_{x\Rightarrow\infty}\frac{x-2}{3x-4}=\frac{\infty-2}{3(\infty)-4}=\frac{\infty}{\infty}$ *meaning less*

$$\Rightarrow \lim\limits_{x\Rightarrow\infty}\frac{\frac{x-2}{x}}{\frac{3x-4}{x}}=\lim\limits_{x\Rightarrow\infty}\frac{1-\frac{2}{x}}{3-\frac{4}{x}}=\frac{1-\frac{2}{\infty}}{3-\frac{4}{\infty}}=\frac{1-0}{3-0}=\frac{1}{3}$$

2- $\lim\limits_{x\Rightarrow\infty}\frac{2x^2+3x-1}{5x^2-2x+7}=\frac{\infty}{\infty}$ *meaning less*

$$\Rightarrow \lim\limits_{x\Rightarrow\infty}\frac{\frac{2x^2+3x-1}{x^2}}{\frac{5x^2-2x+7}{x^2}}=\lim\limits_{x\Rightarrow\infty}\frac{2+\frac{3}{x}-\frac{1}{x^2}}{5-\frac{2}{x}+\frac{7}{x^2}}=\frac{2+0+0}{5-0+0}=\frac{2}{5}$$

L Hopital Rule

Suppose that $f(a)=g(a)=0$ **that** $f'(a)$ **and** $g'(a)$ **exist , and that** $g'(a)\neq 0$

Then $\lim\limits_{x\Rightarrow a}\frac{f(x)}{g(x)}=\frac{f'(a)}{f'(b)}$

Find The Following Limits (By using L hospital Rule)

1- $\lim\limits_{x\Rightarrow\infty}\frac{2x-2}{3x-2}=\frac{\infty-1}{\infty-2}=\frac{\infty}{\infty}$ *meaning less* $\lim\limits_{x\Rightarrow\infty}\frac{2}{3}=\frac{2}{3}$

2- $\lim\limits_{x\Rightarrow 0}\frac{2x^2+3x}{5x^2-2x}=\frac{2(0)^2+3(0)}{5(0)^2-2(0)}=\frac{0}{0}$ *meaning less* $\Rightarrow \lim\limits_{x\Rightarrow 0}\frac{4x+3}{10x-2}=\frac{4(0)+3}{10(0)-2}=\frac{-3}{2}$

3- $\lim\limits_{x\Rightarrow 0}\frac{\sqrt{1+x}-1}{x}=\frac{\sqrt{1+0}-1}{0}=\frac{0}{0}$ *meaning less* $\Rightarrow \lim\limits_{x\Rightarrow 0}\frac{1/2(1+x)^{-1/2}}{10x-2}=\frac{1}{2}(1+0)^{-1/2}=\frac{1}{2}$

4- $\lim\limits_{x\Rightarrow 1}\frac{x^3-3x+2}{x^3-x^2-x+1}=\frac{(1)^3-3(1)+2}{(1)^3-(1)^2-(1)+1}=\frac{-2+2}{-1+1}=\frac{0}{0}$ *meaning less*

$$\lim\limits_{x\Rightarrow 1}\frac{3x^2-3}{3x^2-2x-1}=\frac{3-3}{3-2-1}=\frac{0}{0}\Rightarrow \lim\limits_{x\Rightarrow 1}\frac{6x}{6x-2}=\frac{6}{4}=\frac{3}{2}$$

E-mail aljelawy2000@yahoo.com

5- $\lim\limits_{x\Rightarrow\pi/2}[\sec(x)-\tan(x)]=\lim\limits_{x\Rightarrow\pi/2}\left(\frac{1}{\cos(x)}-\frac{\sin(x)}{\cos(x)}\right)\Rightarrow\lim\limits_{x\Rightarrow\pi/2}\frac{1-\sin(x)}{\cos(x)}=\frac{1-\sin(\pi/2)}{\cos(\pi/2)}=\frac{1-1}{0}=\frac{0}{0}$

$\Rightarrow\lim\limits_{x\Rightarrow\pi/2}\frac{-\cos(x)}{-\sin(x)}=\frac{\cos(\pi/2)}{\sin(\pi/2)}=\frac{0}{1}=0$

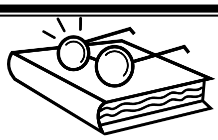
6- $\lim\limits_{x\Rightarrow0}\frac{\sin(x)}{x}=\frac{\sin(0)}{0}=\frac{0}{0}$ *meaning less* $\Rightarrow\lim\limits_{x\Rightarrow0}\frac{\cos(x)}{1}=\frac{\cos(0)}{1}=\frac{1}{1}=1$

7- $\lim\limits_{x\Rightarrow0}\frac{\ln(x+1)-2x}{x^2}=\frac{\ln(1+0)-2(0)}{(0)^2}=\frac{\ln(1)-0}{0}=\frac{0}{0}$ *meaning less* $\lim\limits_{x\Rightarrow0}\frac{\frac{1}{(x+1)}-2}{2x}=\frac{\frac{1}{0+1}-2}{2(0)}=\frac{-1}{0}=\infty$

8- $\lim\limits_{x\Rightarrow0}\frac{e^{2x}-2x-1}{1-\cos(x)}=\frac{e^0-0-1}{1-\cos(0)}=\frac{1-1}{1-1}=\frac{0}{0}$ *meaning less*

$\Rightarrow\lim\limits_{x\Rightarrow0}\frac{2e^{2x}-2}{\sin(x)}=\frac{2e^0-2}{\sin(0)}=\frac{2-2}{0}=\frac{0}{0}$ *meaning less* $\Rightarrow\lim\limits_{x\Rightarrow0}\frac{4e^{2x}}{\cos(x)}=\frac{4e^0}{\cos(0)}=\frac{4}{1}=4$

9- $\lim\limits_{x\Rightarrow0}\frac{\sin(2x)}{\sin(3x)}=\frac{\sin(0)}{\sin(0)}=\frac{0}{0}$ *meaning less* $\lim\limits_{x\Rightarrow0}\frac{2\cos(2x)}{3\cos(3x)}=\frac{2\cos(0)}{3\cos(0)}=\frac{2}{3}$



How To Solve

1	$\lim\limits_{x\Rightarrow0}\frac{x-2}{x^2-4}$	$\lim\limits_{t\Rightarrow\infty}\frac{6t+5}{3t-8}$	$\lim\limits_{x\Rightarrow\pi/2}\frac{2x-\pi}{\cos(x)}$	$\lim\limits_{t\Rightarrow\infty}\frac{t^2-2t}{3t^2-2}$
2	$\lim\limits_{x\Rightarrow\pi/3}\frac{\cos(x)-1/2}{x-\pi/3}$	$\lim\limits_{\theta\Rightarrow\pi}\frac{\sin(\theta)}{\pi-\theta}$	$\lim\limits_{x\Rightarrow1}\frac{x^3-1}{4x^3-x-3}$	$\lim\limits_{t\Rightarrow0}\frac{\cos(t)-1}{t^2}$

<u>Note</u>

$$\log_b(x)\Leftrightarrow x=b^y\Rightarrow b=\begin{cases}0 & \text{if } b=0 \text{ we write } \log_{10}(x)=\log(x)\\ 2.817=e & \text{if } b=e \text{ we write } \log_e(x)=\ln(x)\end{cases}$$

(Real Natural Logarithm)

$1-\sinh(0)=\frac{e^0-e^{-0}}{2}=\frac{1-1}{2}=0$

$2-\sin(30)=\sin(\frac{\pi}{6})=\frac{1}{2}$

$3-\sin(90)=\sin(\frac{\pi}{2})=1$

$4-\ \theta=\tan^{-1}(60)\ \Rightarrow\ \theta=\tan(60)\Rightarrow\theta=$

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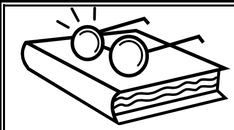


How To Solve

Show That

1	$\sin^2(\theta)+\cos^2(\theta)=1$	$\sinh(x)+\cosh(x)=e^x$	$\tan(a+b)=\dfrac{\tan(a)+\tan(b)}{1-\tan(a)\tan(b)}$	$\sin(a-\frac{\pi}{2})=-\cos(x)$

Derivative

1 $y^4+\tan(xy)+\sin(x)=2$ $\qquad$ $y=\tan\left[\sec^3\left(x^2+\sin(2x)\right)\right]^{-2}$

2 $y=\sinh^2\left[\sec^{-1}(2x)\right].\cos^{-1}(\frac{1}{x})$ $\qquad$ $y=\left[\ln(x^2+\sin^{-1}(2x)\right]^3$

3 $y=\sin^4[\ln(x)]\tan^{-1}(x^2+1)$ $\qquad$ $y=e^{2\sin(3x)}.\csc^{-1}(x^2+2x+1)$

4 $y=2^{\sin^{-1}(3x+1)}.e^{x^2+1}$ $\qquad$ $y=\tanh\left|x^2+\sin(2x)\right|$

5 $y=\tan^{-1}[\sin(x)].\tanh^3(e^{2x})$ $\qquad$ $y=\sinh^2\left|\sin^3(x^2+2x+3)\right|$

6 $y=\dfrac{\ln(x^2+1),\sec(x+2).e^{x^2+1}}{2^x(x+1)^2\tan^{-1}(e^x)}$ $\qquad$ $y=\operatorname{sec}h^{-2}\left|\operatorname{sec}h(x^2+1)\right|\sin^{-1}[x^2+\operatorname{csc}h(x)]$

Solve for x

1 $3^x=2^{x+1}$, $\ln(x-1)-\ln(x)=2y$

Simplify

$e^{\ln[\sin(x)]}$, $e^{x+\ln(x)}$, $\ln(e^{x^2})$

Find the domain & Range

1 $y=\dfrac{1}{x^2-4}$, $y=\sqrt{x-1}$, $y=\sin(x)$

E-mail aljelawy2000@yahoo.com

IF $y=\sin^{-1}(x^2+1)$ Find $\dfrac{dy^3}{d\sec(2x)}$

Solution:- **Let** $u=y^3$, $v=\sec(2x)$

$$y=\sin^{-1}(x^2+1) \Rightarrow \frac{dy}{dx}=\frac{2x}{\sqrt{1-(x^2+1)^2}}$$

$$u=y^3 \Rightarrow \frac{du}{dy}=3y^2$$

قلب المشتقه

$$v=\sec(2x) \Rightarrow \frac{dv}{dx}=2\sec(2x)\tan(2x) \Rightarrow \frac{dx}{dv}=\frac{1}{2\sec(2x)\tan(2x)}$$

$$\frac{dy^3}{d\sec(2x)}=\frac{du}{dv}=\frac{du}{dy}.\frac{dy}{dx}.\frac{dx}{dv}=(3y^2).\frac{1}{2\sec(2x)\tan(2x)}.\frac{2x}{\sqrt{1-(x^2+1)^2}}$$

IF $y=\sin^{-1}(t)$, $x=\cos^{-1}(t)$ **Find** $\dfrac{dy}{dx}, \dfrac{d^2y}{dx^2}$

$$y=\sin^{-1}(t) \Rightarrow \frac{dy}{dt}=\frac{1}{\sqrt{1-t^2}}$$

$$y=\cos^{-1}(t) \Rightarrow \frac{dx}{dt}=\frac{-1}{\sqrt{1-t^2}}$$

$$\Rightarrow \frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=-1 \qquad \frac{d^2y}{dx^2}=0$$

Solve for y **If** $\ln(y-1)=x+\ln(x)$

Solution:- $e^{\ln(y-1)}=e^{x+\ln(x)} \Rightarrow y-1=e^{x+\ln(x)}=e^x.e^{\ln(x)}=xe^x \Rightarrow y=xe^x+1$

The Function $a^{u(x)}$ **IF** $y=a^{u(x)} \Rightarrow \dfrac{dy}{dx}=a^{u(x)}.\ln(a).\dfrac{du}{dx}$

Find $\dfrac{dy}{dx}$

1- $y=2^{x^2+1} \Rightarrow \dfrac{dy}{dx}=2^{x^2+3}.\ln(2).(2x)$

2- $y=4^{\sin(x)} \Rightarrow \dfrac{dy}{dx}=4^{\sin(x)}.\ln(4).\cos(x)$

3- $y=2^{\sin^{-1}(x)}.e^{x+1} \Rightarrow \dfrac{dy}{dx}=2^{\sin^{-1}(x)}.e^{x+1}+e^{x+1}.2^{\sin^{-1}(x)}.\ln(2).\dfrac{1}{\sqrt{1-x^2}}$

E-mail aljelawy2000@yahoo.com

Methods Of Integration

Integral Formula (Standard Form)

1	$\int u^n du = \frac{u^{n+1}}{n+1} + c \quad n \neq -1$	$\int \frac{du}{u} = \ln(u) + c$
2	$\int e^u du = e^u + c \qquad e = 2.718$	$\int a^u du = \frac{a^u}{\ln(a)} + c$ **a is constant**
3	$\int \sin(u) du = -\cos(u) + c$	$\int \sinh(u) du = \cosh(u) + c$
4	$\int \cos(u) du = \sin(u) + c$	$\int \cosh(u) du = \sinh(u) + c$
5	$\int \sec^2(u) du = \tan(u) + c$	$\int \sec h^2(u) du = \tanh(u) + c$
6	$\int \csc^2(u) du = -\cot(u) + c$	$\int \csc h^2(u) du = -\coth(u) + c$
7	$\int \sec(u)\tan(u) du = \sec(u) + c$	$\int \sec h(u) \tanh(u) du = -\sec h(u) + c$
8	$\int \csc(u)\cot(u) du = -\csc(u) + c$	$\int \csc h(u) \coth(u) du = -\csc h(u) + c$
9	$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}(\frac{u}{a}) + c$	$\int \frac{du}{\sqrt{u^2 + a^2}} = \sinh^{-1}(\frac{u}{a}) + c$
10	$\int \frac{-du}{\sqrt{a^2 - u^2}} = \cos^{-1}(\frac{u}{a}) + c$	$\int \frac{du}{\sqrt{u^2 - a^2}} = \cosh^{-1}(\frac{u}{a}) + c$
11	$\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}(\frac{u}{a}) + c$	$\int \frac{du}{a^2 - u^2} = \begin{cases} \frac{1}{a}\tanh^{-1}(\frac{u}{a}) + c & \lvert u \rvert \prec a \\ \frac{1}{a}\coth^{-1}(\frac{u}{a}) + c & \lvert u \rvert \succ a \end{cases}$
12	$\int \frac{-du}{a^2 + u^2} = \frac{1}{a}\cot^{-1}(\frac{u}{a}) + c$	
13	$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}(\frac{u}{a}) + c$	$\int \frac{du}{u\sqrt{a^2 - u^2}} = -\frac{1}{a}\sec h^{-1}(\frac{u}{a}) + c$
14	$\int \frac{-du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\csc^{-1}(\frac{u}{a}) + c$	$\int \frac{du}{u\sqrt{a^2 + u^2}} = \frac{1}{a}\csc h^{-1}(\frac{u}{a}) + c$

E-mail aljelawy2000@yahoo.com

Method [1]

Integration By Substitution

The goal of this method is to transform the integral into a standard from

To evaluate the integral $I=\int f[g(x)]\,g'(x)dx$ **carry out the following steps**

1- substitute $u=g(x)$ **the** $du=g'(x)dx$ **to obtain** $I=\int f(u)du$

2- Evaluate $I=\int f(u)du$ **by integrating w.r.t** u

3- Replace u **by** $g(x)$ **in the final result**

Evaluate $I=\int \frac{dx}{\sqrt[3]{1-2x}}$

Solution :- $I=\int (1-2x)^{-\frac{1}{3}}dx$ **Let** $u=1-2x \Rightarrow du=-2dx \Rightarrow dx=\frac{du}{-2}$

$$I=\int (1-2x)^{-\frac{1}{3}}dx \Rightarrow I=\int u^{-\frac{1}{3}}\frac{du}{-2}=\frac{-1}{2}\int u^{-\frac{1}{3}}du=\frac{-1}{2}\frac{u^{\frac{2}{3}}}{\frac{2}{3}}+c=\frac{-3}{4}(1-2x)^{\frac{2}{3}}+c$$

Evaluate $I=\int \sin^2(5x)\cos(5x)dx$

Solution :- **Let** $u=\sin(5x) \Rightarrow du=5\cos(5x)dx \Rightarrow dx=\frac{du}{5\cos(5x)}$

$$I=\int \sin^2(5x)\cos(5x)dx \Rightarrow I=\int u^2\cos(5x)\frac{du}{5\cos(5x)}=\frac{1}{5}\int u^2du=\frac{1}{5}\frac{u^3}{3}+c=\frac{1}{15}[\sin(5x)]^3+c$$

Evaluate $I=\int xe^{x^2+1}dx$

Solution :- **Let** $u=x^2+1 \Rightarrow du=2xdx \Rightarrow dx=\frac{du}{2x}$

E-mail *aljelawy2000@yahoo.com*

$$I=\int xe^{x^2+1}dx \Rightarrow I=\int xe^{u}\frac{du}{2x}=\frac{1}{2}\int e^{u}du=\frac{1}{2}e^{u}+c=\frac{1}{2}e^{x^2+1}+c$$

Evaluate $I=\frac{1}{3}\int\frac{3\cos(3x)}{4+\sin(3x)}dx$

Solution :- **Let** $u=4+\sin(3x) \Rightarrow du=3\cos(3x)dx \Rightarrow dx=\frac{du}{3\cos(3x)}$

$$I=\frac{1}{3}\int\frac{3\cos(3x)}{4+\sin(3x)}dx \Rightarrow I=\frac{1}{3}\int\frac{3\cos(3x)}{u}\frac{du}{3\cos(3x)}=\frac{1}{3}\int\frac{1}{u}du=\ln(u)+c=\frac{1}{3}\ln[4+\sin(3x)]+c$$

Evaluate $I=\frac{1}{3}\int\frac{3\cos(3x)}{4+\sin^2(3x)}dx$

Solution :- **Let** $u=\sin(3x) \Rightarrow du=3\cos(3x)dx \Rightarrow dx=\frac{du}{3\cos(3x)}$

$$\Rightarrow I=\frac{1}{3}\int\frac{3\cos(3x)}{2^2+u^2}\frac{du}{3\cos(3x)}=\frac{1}{3}\int\frac{1}{2^2+u^2}du=\frac{1}{3}\frac{1}{a}\tan^{-1}(\frac{u}{a})+c=\frac{1}{3}\frac{1}{2}\tan^{-1}(\frac{u}{2})+c$$

Evaluate $I=\int\frac{dx}{\sqrt{4-9x^2}}$

Solution :- $I=\int\frac{dx}{\sqrt{4-(3x)^2}}$ **Let** $u=3x \Rightarrow du=3dx \Rightarrow dx=\frac{du}{3}$

$$\Rightarrow I=\int\frac{1}{\sqrt{2^2-u^2}}\frac{du}{3}=\frac{1}{3}\int\frac{1}{\sqrt{a^2-u^2}}du=\frac{1}{3}\sin^{-1}(\frac{u}{a})+c=\frac{1}{3}\sin^{-1}(\frac{u}{2})+c=\frac{1}{3}\sin^{-1}(\frac{3x}{2})+c$$

Evaluate $I=\int\frac{\cos(x)dx}{\sin^2(x)}$ **Let** $u=\sin(x) \Rightarrow du=\cos(x)dx \Rightarrow dx=\frac{du}{\cos(x)}$

Solution $I=\int\frac{\cos(x)dx}{\sin^2(x)}=\int\frac{\cos(x)}{u^2}\frac{du}{\cos(x)}=\int u^{-2}du=\frac{u^{-2+}}{-2+1}+c$

Evaluate $I=\int\tan^3(3x)3\sec^2(3x)dx$

Solution :- **Let** $u=\tan(3x) \Rightarrow du=3\sec^2(3x)dx \Rightarrow dx=\frac{du}{3\sec^2(3x)}$

E-mail *aljelawy2000@yahoo.com*

$$I=\int \tan^3(3x)3\sec^2(3x)dx \Rightarrow \quad I=\int u^3 3\sec^2(3x)\frac{du}{3\sec^2(3x)}=\int u^3du=\frac{u^4}{4}+c=\frac{1}{4}[\tan(3x)]^4+c$$

Evaluate $I=\int \frac{\sin^2(2x)}{1+\cos(2x)}dx$

Solution :- $I=\int \frac{\sin^2(2x)}{1+\cos(2x)}dx=\int \frac{1-\cos^2(2x)}{1+\cos(2x)}dx=\int \frac{(1-\cos 2x)(1+\cos 2x)}{1+\cos(2x)}$

$$\Rightarrow \int [1-\cos(2x)]dx = x-\frac{1}{2}\sin(2x)+c$$

Evaluate $I=\int \frac{\sqrt{x}}{4+x^3}dx$

Solution :- $I=\int \frac{\sqrt{x}}{4+\left((x)^{\frac{3}{2}}\right)^2}dx$ **Let** $u=x^{\frac{3}{2}} \Rightarrow du=\frac{3}{2}x^{\frac{1}{2}}dx=\frac{3}{2}\sqrt{x}\ dx$

$$\Rightarrow dx=\frac{du}{\frac{3}{2}\sqrt{x}} \Rightarrow \int \frac{\sqrt{x}}{2^2+(u)^2}\frac{du}{\frac{3}{2}\sqrt{x}}=\frac{2}{3}\int \frac{du}{a^2+(u)^2}=\frac{2}{3}\frac{1}{2}\tan^{-1}\left(\frac{u}{2}\right)=\frac{1}{3}\tan^{-1}\left(\frac{x^{3/2}}{2}\right)+c$$

Evaluate $I=\int \sec^2(x)dx$

Solution :- $\Rightarrow I=\int \sec^2(x)dx=\tan(x)+c$

Evaluate $I=\int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}dx$

Solution :- **Let** $u=\sin^{-1}(x) \Rightarrow du=\frac{1}{\sqrt{1-x^2}}dx \Rightarrow dx=\sqrt{1-x^2}du$

E-mail aljelawy2000@yahoo.com

$$I=\int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}dx=\int \frac{u}{\sqrt{1-x^2}}\sqrt{1-x^2}du \Rightarrow I\int udu=\frac{u^2}{2}+c=\frac{[\sin^{-1}(x)]^2}{2}+c$$

Evaluate $I=\int \frac{e^x}{1+e^{2x}}dx \Rightarrow \int \frac{e^x}{1+(e^x)^2}dx$

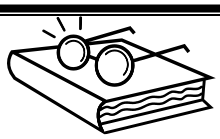
Solution :- **Let** $u=e^x \Rightarrow du=e^x dx \Rightarrow dx=\frac{du}{e^x}$

$$I=\int \frac{e^x}{1+(e^x)^2}dx=\int \frac{\cancel{e^x}}{1+(u)^2}\frac{du}{\cancel{e^x}} \Rightarrow I=\int \frac{du}{1+u^2}=\tan^{-1}(u)+c=\tan^{-1}(e^x)+c$$

Evaluate $I=\int \frac{[\ln(x)]^2}{x}dx$

Solution :- **Let** $u=\ln(x) \Rightarrow du=\frac{1}{x}dx \Rightarrow dx=xdu$

$$I=\int \frac{[u]^2}{x}xdx=\int u^2du \Rightarrow I=\frac{u^3}{3}=\frac{[\ln(x)]^3}{3}+c$$



How To Solve

1	$\int (x-\frac{1}{x})^2$	$\int x.2^{x^2+3}dx$	$\int \frac{\sec^2(x)}{1+\tan^2(x)}dx$	$\int \frac{e^x}{1+e^{2x}}dx$
2	$\int \tan^2(3x)dx$	$\int \tan(4x)dx$	$\int \frac{dx}{x[1+(\ln(x))^2]}$	$\int \frac{[\ln(x)]^3}{x}$
3	$\int \frac{\sec^2[\ln(x)]}{x}dx$	$\int \frac{2\sin(\sqrt{x})}{\sqrt{x}\sec(\sqrt{x})}dx$	$\int \frac{dx}{\sqrt{x}(1+x)}$	$\int \sin^2(x)dx$
4	$\int_4^9 \frac{dx}{x-\sqrt{x}}$	$\int_0^\pi \sin^3(x)\cos(x)dx$	$\int_0^\infty e^{-x-e^{-x}}dx$	$\int \tan(x)\frac{\ln[\cos(x)]}{2}dx$

E-mail aljelawy2000@yahoo.com

Method [2]

Certain Power Of Trigonometric

Consider the following integrals forms

$\mathscr{A}$	$\int \sin^m(u)\cos^n(u)\,du$	$\int \sinh^m(u)\cosh^n(u)\,du$
$\mathscr{B}$	$\int \tan^m(u)\sec^n(u)\,du$	$\int \tanh^m(u)\operatorname{sech}^n(u)\,du$
$\mathscr{C}$	$\int \cot^m(u)\csc^n(u)\,du$	$\int \coth^m(u)\operatorname{csch}^n(u)\,du$

Under Type ($\mathscr{A}$) There are three cases

Case ( )

If (m) is odd and (+ ive) , We factor out $[\sin(u)\ \sinh(u)]$ **and change the remaining even power of** $[\sin(u)\ \sinh(u)]$ **to** $[\cos(u)\ \cosh(u)]$ **using the identities**

$$\sin^2(u) = 1 - \cos^2(u) \qquad , \qquad \sinh^2(u) = \cosh^2(u) - 1$$

Evaluate $I = \int \sin^5(2x)\cos^{\frac{-3}{2}}(2x)\,dx$

Solution :- $\Rightarrow I = \int \sin^4(2x)\cos^{\frac{-3}{2}}(2x)\ \sin(2x)dx$

$$\Rightarrow I = \int \left(1-\cos^2(2x)\right)^2 \cos^{\frac{-3}{2}}(2x)\ \ \sin(2x)\,dx = \int \left(1-2\cos^2(2x)+\cos^4(2x)\right)\cos^{\frac{-3}{2}}(2x)\ \sin(2x)\,dx$$

$$\Rightarrow \int \left(\cos^{\frac{-3}{2}}(2x) - 2\cos^{\frac{1}{2}}(2x) + \cos^{\frac{5}{2}}(2x)\right)\sin(2x)dx$$

E-mail aljelawy2000@yahoo.com

$$\Rightarrow \int\left(\cos^{\frac{-3}{2}}(2x)\sin(2x) - 2\cos^{\frac{1}{2}}(2x)\sin(2x) + \cos^{\frac{5}{2}}(2x)\ \sin(2x) \right)dx$$

$$\Rightarrow \quad = -\frac{1}{2}\frac{\cos^{\frac{-1}{2}}(2x)}{\frac{-1}{2}} + \frac{1}{2}\frac{2\cos^{3}(2x)}{\frac{3}{2}} - \frac{1}{2}\frac{\cos^{\frac{7}{2}}(2x)}{\frac{7}{2}} + c$$

Case ( )

If (n) is odd and (+ ive) , We factor out $[\cos(u)\ \cosh(u)]$ and change the remaining even power of $[\cos(u)\ \cosh(u)]$ to $[\sin(u)\ \sinh(u)]$ using the identities

$$\cos^2(u) = 1 - \sin^2(u) \quad , \quad \cosh^2(u) = 1 + \sinh^2(u)$$

Evaluate $I = \int \sin^4(3x)\cos^3(3x)\,dx$

Solution :- $\Rightarrow I = \int \cos^2(3x)\ \sin^4(3x)\ \ \cos(3x)dx = \int \left(1 - \sin^2(3x)\right)\sin^4(3x)\ \cos(3x)dx$

$$\Rightarrow = \int\left(\sin^4(3x) - \sin^6(3x)\right)\cos(3x)dx = \int \sin^4(3x)\cos(3x)dx - \int \sin^6(3x)\ \cos(3x)dx$$

Case ( )

If both (n) and (m) are even and (+ ive) , (or one of them zero) we reduce the degree of the expression by using the identities

$$\sin^2(u) = \frac{1-\cos(2u)}{2} \quad , \quad \sinh^2(u) = \frac{\cosh(2u)-1}{2}$$

$$\cos^2(u) = \frac{1+\cos(2u)}{2} \quad , \quad \cosh^2(u) = \frac{\cosh(2u)+1}{2}$$

Evaluate $I = \int \sin^2(2x)\cos^2(2x)dx$

Solution :- $\Rightarrow \frac{1}{4}\int(1-\cos(4x))(1+\cos(4x))dx = \frac{1}{4}\int\left(1-\cos^2(4x)\right)dx$

$$\Rightarrow \frac{1}{4}\int\left(1 - \frac{1+\cos(8x)}{2}\right)dx = \frac{1}{4}\int\left(\frac{1}{2} - \frac{1}{2}\cos(8x)\right)dx = \frac{1}{4}\left[\frac{x}{2} - \frac{1}{16}\sin(8x)\right] + c$$

Under Type ($\mathscr{B}$) There are two cases

E-mail aljelawy2000@yahoo.com

Case ( )

If (n) is even and (+ ive) , We factor out $\left|\sec^2(u)\ \sec^2 h(u)\right|$ **and change the remaining even power of** $[\sec(u)\ \sec h(u)]$ **to** $[\tan(u)\ \tanh(u)]$ **using the identities**

$$\sec^2(u) = 1+\tan^2(u) \quad , \qquad \sec h^2(u) = 1-\tanh^2(u)$$

Evaluate $I = \int \sec^4(x)\tan^{-\frac{1}{3}}(x)\,dx$

Solution :- $\Rightarrow \int \sec^2(x)\tan^{-\frac{1}{3}}(x)\ \sec^2(x)\,dx = \int \left(1+\tan^2(x)\right)\tan^{-\frac{1}{3}}(x)\sec^2(x)dx$

$$\Rightarrow \int \left(\tan^{-\frac{1}{3}}(x) - \tan^{\frac{5}{3}}(x)\right)\sec^2(x)\,dx = 2\left[\frac{\tan^{\frac{2}{3}}(x)}{2/3} - \frac{\tan^{\frac{8}{3}}(x)}{8/3}\right] + c$$

Case ( )

If (m) is odd and (+ ive) , We factor out $[\sec(u)\tan(u)\{\sec h(u)\tan(u)\}]$ **and change the remaining even power of** $[\sec(u)\ \sec h(u)]$ **to** $[\tan(u)\ \tanh(u)]$ **using the identities**

$$\tan^2(u) = \sec^2(u)-1 \quad , \qquad \tanh^2(u) = 1-\sec h^2(u)$$

Evaluate $I = \int \tan^3(2x)\sec^{-\frac{1}{4}}(2x)\,dx$

Solution :- $I = \int \left(\sec^2(2x)-1\right)\ \sec^{-\frac{5}{4}}(2x)\ \ \sec(2x)\tan(2x)\,dx$

$$\Rightarrow \int \left(\sec^{\frac{3}{4}}(2x) - \sec^{-\frac{5}{4}}(2x)\right)\sec(2x)\tan(2x)\,dx = \frac{1}{2}\left[\frac{\sec^{\frac{7}{4}}(2x)}{7/4} - \frac{\sec^{-\frac{1}{4}(2x)}}{-1/4}\right] + c$$

Under Type ($\mathscr{C}$ **) There are two cases similar to there of type (** $\mathscr{B}$ **) where the identities :**

$$\csc^2(u) = \cot^2(u)+1 \quad , \quad \csc h^2(u) = \coth^2(u)-1$$

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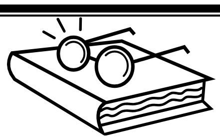
Evaluate $I = \int \cot^3(x)\csc^4(x)\,dx$

Solution :-



How To Solve

1	$\int \sin^5(2x)\,dx$	$\int \csc^2(x)\,dx$	$\int \cos^2(x)\,dx$	$\int \cot^4(3x)dx$
2	$\int \tan^3(x)\sec(x)\,dx$	$\int \cot^3(2x)\csc^4(2x)dx$	$\int \sin^3(2x)\,dx$	$\int \cos^3(x)\sin^{\frac{1}{2}}(x)\,dx$
3	$\int \sin^2(x)\cos^2(x)\,dx$	$\int \tan(\frac{x}{2})\sec^2(\frac{x}{2})\,dx$	$\int \sin^4(x)\cos^{-2}(x)\,dx$	$\int \sin(x)dx$

Method [3]

Trigonometric Substitutions

If the integral involve one of the forms $\left(a^2+u^2, \sqrt{a^2-u^2}, \sqrt{a^2+u^2}, \sqrt{u^2-a^2}\right)$ then the substitutions as follows :

1 - If $\sqrt{a^2-u^2}$ Let $u = a\sin(\theta) \Rightarrow a^2-u^2 = a^2\cos^2(\theta)$

2 - If $\sqrt{a^2+u^2}$, a^2+u^2 Let $u = a\tan(\theta) \Rightarrow a^2+u^2 = a^2\sec^2(\theta)$

3 - If $\sqrt{u^2-a^2}$ Let $u = a\sec(\theta) \Rightarrow u^2-a^2 = a^2\tan^2(\theta)$

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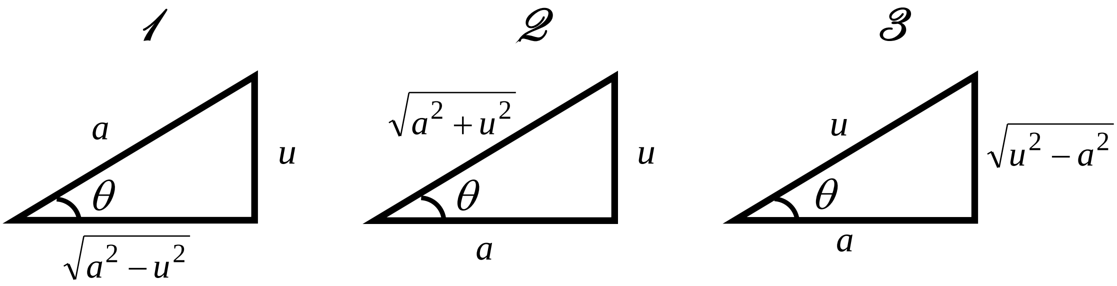

Evaluate $I = \int \frac{dx}{4+x^2} dx$

Solution :- $I = \int \frac{dx}{4+x^2} dx = \frac{1}{2}\tan^{-1}(\frac{x}{2}) + c$

Let $x = 2\tan(\theta) \;\Rightarrow\; \tan(\theta) = \frac{x}{2} \;\Rightarrow \theta = \tan^{-1}(\frac{x}{2}) \;\Rightarrow dx = 2\sec^2(\theta)d\theta$

$$I = \int \frac{2\sec^2(\theta)}{4+4\tan^2(\theta)} d\theta = \int \frac{2\sec^2(\theta)}{4\sec^2(\theta)} d\theta = \frac{1}{2}\int d\theta = \frac{1}{2}\theta + c = \frac{1}{2}\tan^{-1}(\frac{x}{2}) + c$$

Evaluate $I = \int_{-1/2}^{\sqrt{3}/2} \sqrt{1-x^2}\,dx$

Solution :- $\Rightarrow \quad x = \sin(\theta) \qquad At \;\; x = -\frac{1}{2} \;\Rightarrow\; -\frac{1}{2} = \sin(\theta) \;\Rightarrow\; \theta = -\frac{\pi}{6}$

$dx = \cos(\theta) \qquad At \;\; x = \frac{\sqrt{3}}{2} \;\Rightarrow\; \frac{\sqrt{3}}{2} = \sin(\theta) \;\Rightarrow\; \theta = \frac{\pi}{3}$

$$I = \int_{-\pi/6}^{\pi/3} \sqrt{1-\sin^2(\theta)}\;\cos(\theta)d\theta = \int_{-\pi/6}^{\pi/3} \cos^2(\theta)d\theta = \int_{-\pi/6}^{\pi/3} \frac{1+\cos(2\theta)}{2} d\theta$$

$$\Rightarrow = \left[\theta + \frac{1}{2}\sin(2\theta)\right]_{-\frac{\pi}{6}}^{\frac{\pi}{3}} = \frac{1}{2}\left[[\frac{\pi}{3} + \frac{1}{2}\sin(\frac{2\pi}{3})] - [\frac{\pi}{6} + \frac{1}{2}\sin(-\frac{\pi}{3})]\right] = \frac{\pi + \sqrt{3}}{4}$$

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Evaluate $I = \int \frac{\sqrt{x^2 - 7}}{x} dx$

Solution :- $x = \sqrt{7}\sec(\theta) \Rightarrow \sec(\theta) = \frac{x}{\sqrt{7}} \Rightarrow \theta = \sec^{-1}(\frac{x}{\sqrt{7}}) \Rightarrow dx = \sqrt{7}\sec(\theta)\tan(\theta)d\theta$

$$\Rightarrow I = \int \frac{\sqrt{7\sec^2(\theta) - 7}}{\sqrt{7}\sec(\theta)}.\sqrt{7}\sec(\theta)\tan(\theta)d\theta = \int \sqrt{7}\tan^2(\theta)d\theta = \sqrt{7}\int(\sec^2(\theta) - 1)d\theta$$

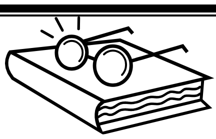
$$\Rightarrow \quad = \sqrt{7}[\tan(\theta) - \theta] + c = \sqrt{7}\left(\tan[\sec^{-1}(\frac{x}{\sqrt{7}})] - \sec^{-1}(\frac{x}{\sqrt{7}})\right) + c$$

Evaluate $I = \int x^3\sqrt{9 + x^2}\, dx$

Solution :- $x = 3\tan(\theta) \quad \Rightarrow dx = 3\sec^2(\theta)\,d\theta$

$$\Rightarrow I = \int 27\tan^3(\theta)\; 3\sec(\theta)\; 3\sec^2(\theta)\; d\theta = (27)(9)\int \tan^3(\theta)\sec^3(\theta)d\theta$$

$$\Rightarrow I = 243\int \left(\sec^2(\theta) - 1\right)\sec^2(\theta)\; \sec(\theta)\tan(\theta)d\theta = \int\left(\sec^4(\theta) - \sec^2(\theta)\right)\; \sec(\theta)\tan(\theta)d\theta$$



How To Solve

1	$\int_{-6}^{-2\sqrt{3}} \frac{dx}{x\sqrt{x^2 - 9}}$	$\int_0^2 \frac{x^2 dx}{x^2 + 4}$	$\int_0^{2\sqrt{3}} \frac{x^3 dx}{\sqrt{x^2 + 4}}$	$\int_0^{\sqrt{5}} x^2\sqrt{5 - x^2}dx$
2	$\int \frac{\sqrt{4x^2 - 9}}{x}dx$	$\int \frac{dx}{(9 - x^2)^{\frac{3}{2}}}dx$	$\int \frac{\sin(x)}{\sqrt{2 - \cos^2(x)}}dx$	$\int \frac{dx}{(x^2 + 4)^2}$

E-mail *aljelawy2000@yahoo.com*

Method [4]

Integral Involving Quadratic Function

If the integral involve a quadratic function $\left(x^2+ax+b\right)$, We reduce it to the from $\left(u^2+B\right)$ by completing the square as follows:

$$\left(x^2+ax+b\right)=\left(x^2+ax+\frac{a^2}{4}+b-\frac{a^2}{4}\right)=\left(x+\frac{a}{2}\right)^2+\left(b-\frac{a^2}{4}\right)=u^2+b \quad \text{where } u=x+\frac{a}{2}$$

And $B=b-\frac{a^2}{4}$ then the solution can be found by method [3]

Evaluate $I=\int \frac{dx}{\sqrt{2x-x^2}}dx$

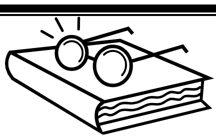
Solution :- $\Rightarrow \; I=\int \frac{dx}{\sqrt{-(x^2-2x+1-1)}}=\int \frac{dx}{\sqrt{-[(x-1)^2-1]}}=\int \frac{dx}{\sqrt{[1-(x-1)^2]}}$

Let $u=x-1 \;\Rightarrow\; du=dx$

$$I=\int \frac{du}{\sqrt{1-u^2}}=\sin^{-1}(u)+c=\sin^{-1}(x-1)+c$$

Evaluate $I=\int \frac{(4x+5)}{(x^2-2x+2)^{3/2}}$

Solution :- $I=\int \frac{(4x+5)dx}{(x^2-2x+1+1)^{3/2}}=\int \frac{(4x+5)dx}{(x-1)^2+1}$



How To Solve

1	$\int_1^2 \frac{dx}{x^2+2x+5}$	$\int_1^2 \frac{3dx}{9x^2-6x+5}$	$\int_{-1}^0 \frac{x^3dx}{\sqrt{3-2x-x^2}}$	$\int \frac{x+3}{x^2+2x+5}$
2	$\int \frac{\cos(x)dx}{\sin^2(x)+2\sin(x)+5}$	$\int \frac{(2x-5)}{\sqrt{4x-x^2}}dx$	$\int \frac{\sqrt{x^2+2x}}{x+2}dx$	$\int \tan(x)dx$

E-mail *aljelawy2000@yahoo.com*

Method [5]

Integration By Parts

Consider

$$w = u.v \quad \Rightarrow \quad dw = u.dv + + v.du \Rightarrow u\,dv = dw - v\,du$$

$$\int u\,dv = \int dw - \int v\,du = w - \int v\,du$$

$$\boxed{\int u\,dv = u.v - \int v.du}$$

Evaluate $I = \int \ln(x)\,dx$

Solution :-

$u = \ln(x) \qquad dv = dx$

$du = \dfrac{dx}{x} \qquad v = x$

$$\Rightarrow I = x\ln(x) - \int x\frac{1}{x}dx = x\ln(x) - \int dx = x\ln(x) - x + c$$

Evaluate $I = \int \tan^{-1}(x)dx$

Solution :-

$u = \tan^{-1}(x) \qquad dv = dx$

$du = \dfrac{dx}{1+x^2} \qquad v = x$

$$\Rightarrow \quad I = x\tan^{-1}(x) - \int \frac{xdx}{1+x^2} = x\tan^{-1}(x) - \frac{1}{2}\ln(1+x^2) + c$$

Evaluate $I = \int xe^x dx$

Solution :-

$u = x \qquad dv = e^x dx$

$du = dx \qquad v = e^x$

$$\Rightarrow \quad I = xe^x - \int e^x\,dx = xe^x - e^x + c$$

E-mail aljelawy2000@yahoo.com

Tabular Integration

Consider the integral of the form $\int f(x)g(x)dx$ **in which** $\int f(x)$ **can be differential repeatedly to Zero and** $g(x)$ **can be integral repeatedly without difficulty Tabular integration save a great deal of work as natural method consider from integration**

$f(x)$ and Its derivative		$g(x)$ and Its Integrals
$f(x)$	+	$g(x)$
$f'(x)$	-	$\int g(x)dx = g_1(x)$
$f''(x)$	+	$\int g_1(x)dx = g_2(x)$
$f'''(x)$		$\int g_2(x)dx = g_3(x)$
$\vdots$		$\vdots$
$f^{n-1}(x)$		
$f^{n}(x) = 0$		$g_{n-1}(x)dx = g_n(x)$

$$I = f(x)g_1(x) - f'(x)g_2(x) + f''(x)g_3(x) - \ldots\ldots\ldots \pm f^{n-1}(x)g_n(x)$$


Evaluate $I = \int x^2 e^x dx$

Solution :-

$f(x)$ and Its derivative		$g(x)$ and Its Integrals
x^2	+	e^x
$2x$	-	$\int e^x dx = e^x$
2	+	$\int e^x dx = e^x$
0		$\int e^x dx = e^x$

$$I = \int x^2 e^x dx = x^2 e^x - 2xe^x + 2e^x + c$$

E-mail aljelawy2000@yahoo.com

Evaluate $I = \int \left(x^3 - 2x^2 + 3x + 1\right)\sin(2x)dx$

Solution :-

$f(x)$ **and Its derivative**		$g(x)$ **and Its Integrals**
$x^3 - 2x^2 + 3x + 1$	+	$\sin(2x)$
$3x^2 - 4x + 3$	-	$\int \sin(2x)dx = -\frac{1}{2}\cos(2x)$
$6x - 4$	+	$\int -\frac{1}{2}\cos(2x)dx = -\frac{1}{4}\sin(2x)$
6	-	$\int -\frac{1}{4}\sin(2x)dx = \frac{1}{8}\cos(2x)$
0		$\int \frac{1}{8}\cos(2x) = \frac{1}{16}\sin(2x)$

$I = \ldots$

Evaluate $I = \int e^x \sin(x)dx$

Solution :- $u = e^x \qquad dv = \sin(x)dx$

$du = e^x dx \qquad v = -\cos(x)$

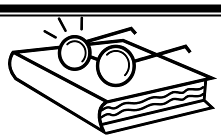
$$I = -e^x\cos(x) + \int e^x \cos(x)dx = -e^x\cos(x) + j$$

Where $u = e^x \qquad dv = \cos(x)dx \qquad j = \int e^x\cos(x)dx \Rightarrow$

$du = e^x dx \qquad v = \sin(x)$

$$j = e^x\sin(x) - \int e^x \sin(x)dx = e^x\sin(x) - I$$

$$I = -e^x\cos(x) + e^x\sin(x) - I \quad \Rightarrow \quad 2I = -e^x\cos(x) + e^x\sin(x) \Rightarrow = \frac{1}{2}e^x\left(\cos(x) - \sin(x)\right)$$



How To Solve

1 $\int x^2\ln(x+1)dx \qquad \int x.\sec^{-1}(x)dx \qquad \int x^2\tan^{-1}(x)dx \qquad \int \sin(\sqrt{2x})dx$

2 $\int \left(x^{-2} + x^{-1} + 1\right)\ln(x)dx \qquad \int \left(x^3 + x^2 + x + 1\right)e^{-2x}dx \qquad \int e^{-x}\sin(x)dx \qquad \int x\sqrt{1-x}$

E-mail aljelawy2000@yahoo.com

3	$\int x^3 e^{-x} dx$	$\int \sqrt{1-x^2}\,\sin^{-1}(x)dx$	$\int x[\ln(x)]^2 dx$	$\int \sin[\ln(x)]dx$

Method [6]

Integration Of Rational Functions

Definition :- A rational function is a quotient of two polynomials as $R(x)=\dfrac{P_n(x)}{Q_m(x)}$ $Q_m \neq 0$ **Where** $P_n(x)$ **and** $Q_m(x)$ **are polynomial of degree** n *and* m

If $n>m$ **we perform a long division until we obtain a rational function whose number numerator degree than or equal to the denominator degree**

Evaluate $I=\int \dfrac{x^5-6x^4-2x^2-3x+4}{x^3+2x+3}dx$

Solution :-

$$x^2-6x-2$$

$$x^3+2x+3 \;\Big)\; x^5-6x^4-2x^2-3x+4$$

$$\mp x^5 \mp 2x^3 \mp 3x^2$$

$$-6x^4-2x^3-5x^2-3x+4$$

$$\pm 6x^4 \qquad \pm 12x^2 \pm 18x$$

$$-2x^3+7x^2+15x+4$$

$$\pm 2x^3 \qquad \pm 4x \pm 6$$

$$7x^2+19x+10$$

$$I=\int\left(x^2-6x-2+\frac{7x^2+19x+10}{x^3+2x+3}\right)dx=\frac{1}{3}x^3-3x^2-2x\int\frac{7x^2+19x+10}{x^3+2x+3}dx$$

If $n \leq m$ **we shall discuss three cases of separating** $\dfrac{P_n(x)}{Q_m(x)}$ **as a sum partial Fractions**

E-mail aljelawy2000@yahoo.com

Case (1) **If the m factors of $Q_m(x)$ are all different and simple , that is $Q_m(x)=(x-a_1)(x-a_2)\ldots\ldots(x-a_{m)}$, then we assign the sum of m partial fractions to these factors as follows :-**

$$\frac{A_1}{(x-a_1)}+\frac{A_2}{(x-a_2)}+\ldots\ldots+\frac{A_m}{(x-a_m)}$$ **where $A_1,A_2,\ldots,A_m$ are constant**

Evaluate $I=\int\frac{x^2+3x+3}{x^3-x}dx=\int\frac{x^2+3x+3}{x(x-1)(x+1)}dx$

Solution :- $\frac{x^2+3x+3}{x(x-1)(x+1)}=\frac{A}{x}+\frac{B}{(x-1)}=\frac{C}{(x+1)}=\frac{A(x-1)(x+1)+Bx(x+1)+cx(x-1)}{x(x-1)(x+1)}$

$$x^2+3x+3=A(x-1)(x+1)+Bx(x+1)+cx(x-1)$$

$$at \quad x=0 \Rightarrow 3=A(0-1)(0+1)+0+0 \quad \Rightarrow A=-3$$
$$at \quad x=1 \Rightarrow 7=0+B(1)(1+1)+0 \quad \Rightarrow B=7/2$$
$$at \quad x=-1 \Rightarrow 1=0+0+C(-1)(-1-1) \quad \Rightarrow C=1/2$$

Or

$$x^2+3x+3=Ax^2-A+Bx^2+Bx+Cx^2-Cx$$
$$=(A+B+C)x^2+(B-C)x-A$$

$$\Rightarrow \left| \begin{array}{l} A+B+C=1 \\ B-C=3 \\ -A=3 \end{array} \right. \quad \Rightarrow A=-3 \quad B=7/2 \quad C=1/2$$

$$I=\int\left(\frac{-3}{x}+\frac{7/2}{x-1}+\frac{1/2}{x+1}\right)dx=-3\ln(x)+\frac{7}{2}\ln(x-1)+\frac{1}{2}\ln(x+1)+$$

Case (2) **Repeated factors of $Q_m(x)$**

Suppose $(x-a)^r$ is the highest power of $(x-a)$ which divided $Q_m(x)$ then to this factor we assign the sum of r partial fractional as follows:-

$$\frac{A_1}{(x-a)}+\frac{A_2}{(x-a)^2}+\ldots\ldots+\frac{A_r}{(x-a)^r}$$ **where $A_1,A_2,\ldots,A_r$ are constant**

Evaluate $I=\int\frac{x^3-3x^2+4x-2}{x(x-1)^2(x+1)(x+2)^3}dx$

E-mail aljelawy2000@yahoo.com

Solution :- $\frac{x^3-3x^2+4x-2}{x(x-1)^2(x+1)(x+2)^3}=\frac{A}{x}+\frac{B}{(x-1)}+\frac{C}{(x-1)^2}+\frac{D}{X+1}+\frac{E}{(x+2)}+\frac{F}{(x+2)^2}+\frac{G}{(x+2)^3}$

Evaluate $I=\int\frac{(x+5)}{(x+2)(x-1)^2}dx$

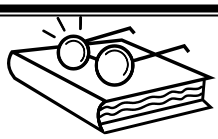
Solution :- $\frac{(x+5)}{(x+2)(x-1)^2}=\frac{A}{(x+2)}+\frac{B}{(x-1)}+\frac{C}{(x-1)^2}=\frac{A(x-1)^2+B(x+2)(x-1)+C(x+2)}{(x+2)(x-1)^2}$

☏ **Case (3)** **Repeated factors of** $Q_m(x)$ **is** (x^2+ax+b) **is not analysis we let** $(ax+b)$

For Example :- $\frac{1}{x^2+1}=\frac{ax+b}{x^2+1}$ **because** (x^2+1) **is not analysis**

Evaluate $I=\int\frac{x}{(x^2+1)(x+1)^2}dx$

Solution :- $\frac{x}{(x^2+1)(x+1)^2}=\frac{Ax+B}{(x^2+1)}+\frac{C}{(x+1)}+\frac{D}{(x+1)^2}$



How To Solve

1	$\int\frac{x^2+3x+4}{x-2}dx$	$\int\frac{x^3-x^2+2x+2}{x^2+3x+2}dx$	$\int\frac{x^4+1}{x^3-x}dx$	$\int\frac{x^2-2}{(x+1)(x-1)^2}dx$
2	$\int\frac{5x^2}{x^2+1}dx$	$\int\frac{x^2+3x+3}{(x+1)(x^2-1)}dx$	$\int\frac{dx}{x^2(x+1)^2}dx$	$\int\frac{x^2-1}{x}dy$

E-mail aljelawy2000@yahoo.com

Method [7]

Integration Of Irrational Function

If the integral contain a single irrational expression the from

$$\sqrt[q]{(ax+b)}=(ax+b)^{\frac{1}{q}} \textbf{ Let } z=(ax+b)^{\frac{1}{q}} \Rightarrow z^q=ax+b \Rightarrow qz^{q-1}=adx \Rightarrow dx=\frac{q}{a}z^{q-1}dz$$

Evaluate $I=\int\frac{2x+3}{\sqrt{x+2}}dx=\int\frac{2x+3}{(x+2)^{1/2}}dx$

Solution :- Let $z=(x+2)^{1/2} \Rightarrow z^2=x+2 \Rightarrow 2zdz=dx$

$$\Rightarrow I=\int\frac{2(z^2-2)+3}{z}\;2zdx=2\int(2z^2-1)dz=2\left(\frac{2}{3}z^3-z\right)+c=2\left(\frac{2}{3}(x+2)^{3/2}-(x+2)^{1/2}\right)+c$$

Evaluate $I=\int\frac{dx}{\sqrt[3]{x^2}+\sqrt{x}}=\int\frac{dx}{x^{2/3}+x^{1/2}}$

Solution :- Let $z=(x)^{1/6} \Rightarrow z^6=x \Rightarrow 6z^5dz=dx$

$$I=\int\frac{6z^5\,dz}{z^4+z^3}=6\int\frac{z^5\,dz}{z^3(z+1)}=6\int\frac{z^2\;dz}{z+1}=6\int(z-1+\frac{1}{z+1})dz$$

$$=6\left(\frac{1}{2}z^2-z+\ln(z+1)\right)+c=6\left(\frac{1}{2}x^{1/3}-x^{1/6}+\ln(x^{1/6}\right)+c$$

Evaluate $I=\int\frac{\sqrt{x}}{1+\sqrt[4]{x}}dx=\int\frac{x^{1/2}}{1+x^{1/4}}dx$

Solution :- Let $z=(x)^{1/4} \Rightarrow z^4=x \Rightarrow 4z^3dz=dx$

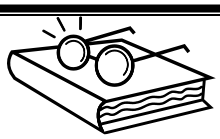
$$I=\int\frac{z^2.4z^3}{1+z}dz=4\int\frac{z^5}{z+1}dz=4\int(z^4-z^3+z^2-z+1-\frac{1}{z+1})dz$$

$$=\left(\frac{1}{5}z^5-\frac{1}{4}z^4+\frac{1}{3}z^3-\frac{1}{2}z^2+z-\ln(z+1)+c\right)$$

E-mail aljelawy2000@yahoo.com

Evaluate $I = \int \frac{\sqrt[3]{x+1}}{x} dx = \int \frac{(x+1)^{1/3}}{x} dx$

Solution :- **Let** $z = (x+1)^{1/3} \Rightarrow z^3 = x+1 \Rightarrow 3z^2 dz = dx$



How To Solve

1	$\int \frac{\sqrt{x}+2}{\sqrt{x}-1} dx$	$\int x\sqrt{x-1}\, dx$	$\int \frac{2x+1}{(x+2)^{2/3}} dx$	$\int x^2 (2x+1)^{-1/3} dx$
2	$\int \frac{2.\sqrt{x+1}-3}{3.\sqrt{x+1}-2} dx$	$\int \frac{dx}{x(1-\sqrt[4]{x})}$	$\int \frac{x}{1+\sqrt{x}+x} dx$	$\int \sqrt{2+\sqrt{x}}\, dx$

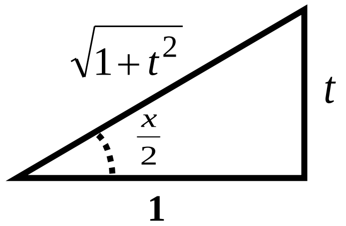
E-mail aljelawy2000@yahoo.com

Method [8]

Integration Of Rational Functions of Trigonometric

If the integral is a rational function of trigonometric substitution of $t=\tan(\frac{x}{2})$ **Will reduce the integral to a relational function of** t **which can be handle by method [6] mathematically speaking**

$$t=\tan(\frac{x}{2}) \;\Rightarrow\; \frac{x}{2}=\tan^{-1}(t) \;\Rightarrow\; \frac{dx}{2}=\frac{dt}{1+t^2} \;\Rightarrow\; \boxed{dx=\frac{2dt}{1+t^2}}$$



$$\sin(\frac{x}{2})=\frac{t}{\sqrt{1+t^2}} \quad , \quad \cos(\frac{x}{2})=\frac{1}{\sqrt{1+t^2}}$$

$$\sin(x)=2\sin(\frac{x}{2})\cos(\frac{x}{2})=\frac{2t}{1+t^2} \quad \Rightarrow \quad \boxed{\sin(x)=\frac{2t}{1+t^2}}$$

$$\cos(x)=\cos^2(\frac{x}{2})-\sin^2(\frac{x}{2})=\frac{1-t^2}{1+t^2} \Rightarrow \boxed{\cos(x)=\frac{1-t^2}{1+t^2}}$$

Evaluate $I=\int\frac{dx}{4-4\cos(x)}dx$

Solution :- $I=\int\frac{dx}{4-4\cos(x)}dx=\int\frac{\frac{2t}{1+t^2}dt}{5-4\left(\frac{1-t^2}{1+t^2}\right)}=\int\frac{2dt}{5(1+t^2)-4(1-t^2)}$

$$=2\int\frac{dt}{1+9t^2}=\frac{2}{3}\int\frac{3dt}{1+(3t)^2}=\frac{2}{3}\tan^{-1}(3t)+c=\frac{2}{3}\tan^{-1}[3\tan(\frac{x}{2})]+c$$

E-mail aljelawy2000@yahoo.com

Evaluate $I=\int \frac{dx}{3\cos(x)+4\sin(x)}$

Solution:-

$$I=\int \frac{\frac{2dt}{1+t^2}}{3\left(\frac{1-t^2}{1+t^2}\right)+4\left(\frac{2t}{1+t^2}\right)}=2\int \frac{dt}{3(1-t^2)+8t}=2\int \frac{dt}{3-3t^2+8t}=-2\int \frac{dt}{3t^2-8t-3}$$

$$I=-2\int \frac{dt}{3t^2-8t-3}=\int \frac{dt}{(3t+1)(t-3)}$$

$$\frac{1}{(3t+1)(t-3)}=\frac{A}{(3t+1)}+\frac{B}{(t-3)} \Rightarrow \frac{1}{(3t+1)(t-3)}=\frac{A(t-3)+B(3t+1)}{(3t+1)(t-3)}$$

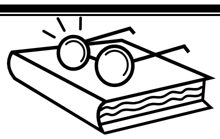
$$1=A(t-3)+B(3t+1) \qquad A=-\frac{3}{10} \qquad B=\frac{1}{10}$$

$$I=-2\int\left(\frac{-3/10}{3t+1}+\frac{1/10}{t-3}\right)dt=-2\left(-\frac{1}{10}\int \frac{3dt}{3t+1}+\frac{1}{10}\int \frac{dt}{t-3}\right)=\frac{1}{5}\ln(3t+1)-\frac{1}{5}\ln(t-3)+c$$

Evaluate $I=\int \frac{\sec^2(x)dx}{1+[\tan(x)]^2}$

Solution:- let $u=\tan(x) \Rightarrow du=\sec^2(x)dx \Rightarrow dx=\frac{du}{\sec^2(x)}$

$$I=\int \frac{\sec^2(x)dx}{1+[\tan(x)]^2}=\int \frac{\sec^2(x)}{1+[u]^2}\cdot\frac{du}{\sec^2(x)}=\int \frac{du}{1+u^2}=\tan^{-1}(u)+c=\tan^{-1}(\tan(x))+c$$

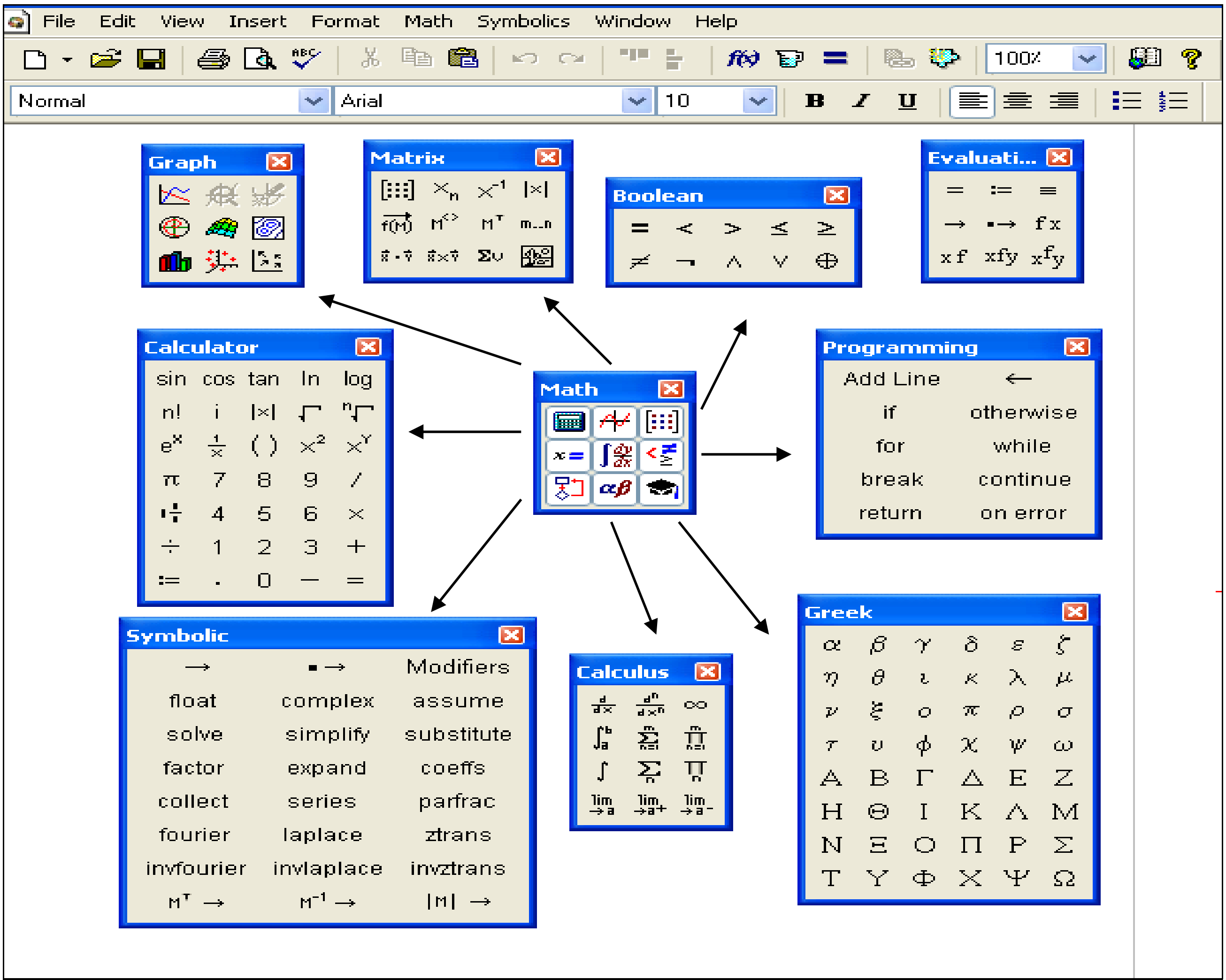


How To Solve

1	$\int \frac{dx}{5+2\cos(x)}$	$\int \frac{dx}{2-\sin(x)}$	$\int \frac{\cos(x)}{5+4\cos(x)}dx$	$\int \frac{dx}{2-\cos(x)}$
2	$\int \frac{dx}{\tan(x)-\sin(x)}$	$\int \frac{dx}{1-\cos(x)}$	$\int \frac{dx}{1+\sin(x)}$	$\int \frac{dx}{\cos(x)+\cot(x)}$

E-mail aljelawy2000@yahoo.com

Mathcad Software



$$\frac{d}{dx}\left(x^5 + \sin(x) + \sqrt{x}\right) \rightarrow 5\cdot x^4 + \cos(x) + \frac{1}{2\cdot x^{\frac{1}{2}}}$$

$$y := \left(\frac{1}{x} + x^2 + e^{2x}\right) \qquad \frac{d}{dx}y \rightarrow \frac{-1}{x^2} + 2\cdot x + 2\cdot \exp(2\cdot x)$$

$$y1 := (\operatorname{asin}(2x)) \qquad \frac{d}{dx}y1 \rightarrow \frac{2}{\left(1 - 4\cdot x^2\right)^{\frac{1}{2}}}$$

$$y2 := e^{x^2+1} + \ln(\sin(x)) \qquad \frac{d}{dx}y2 \rightarrow 2\cdot x\cdot \exp\left(x^2+1\right) + \frac{\cos(x)}{\sin(x)}$$

$$y3 := \left(x^3 + 2x\right)^5$$

$$\frac{d^3}{dx^3}y3 \rightarrow 60\cdot\left(x^3 + 2\cdot x\right)^2\cdot\left(3\cdot x^2 + 2\right)^3 + 360\cdot\left(x^3 + 2\cdot x\right)^3\cdot\left(3\cdot x^2 + 2\right)\cdot x + 30\cdot\left(x^3 + 2\cdot x\right)^4$$

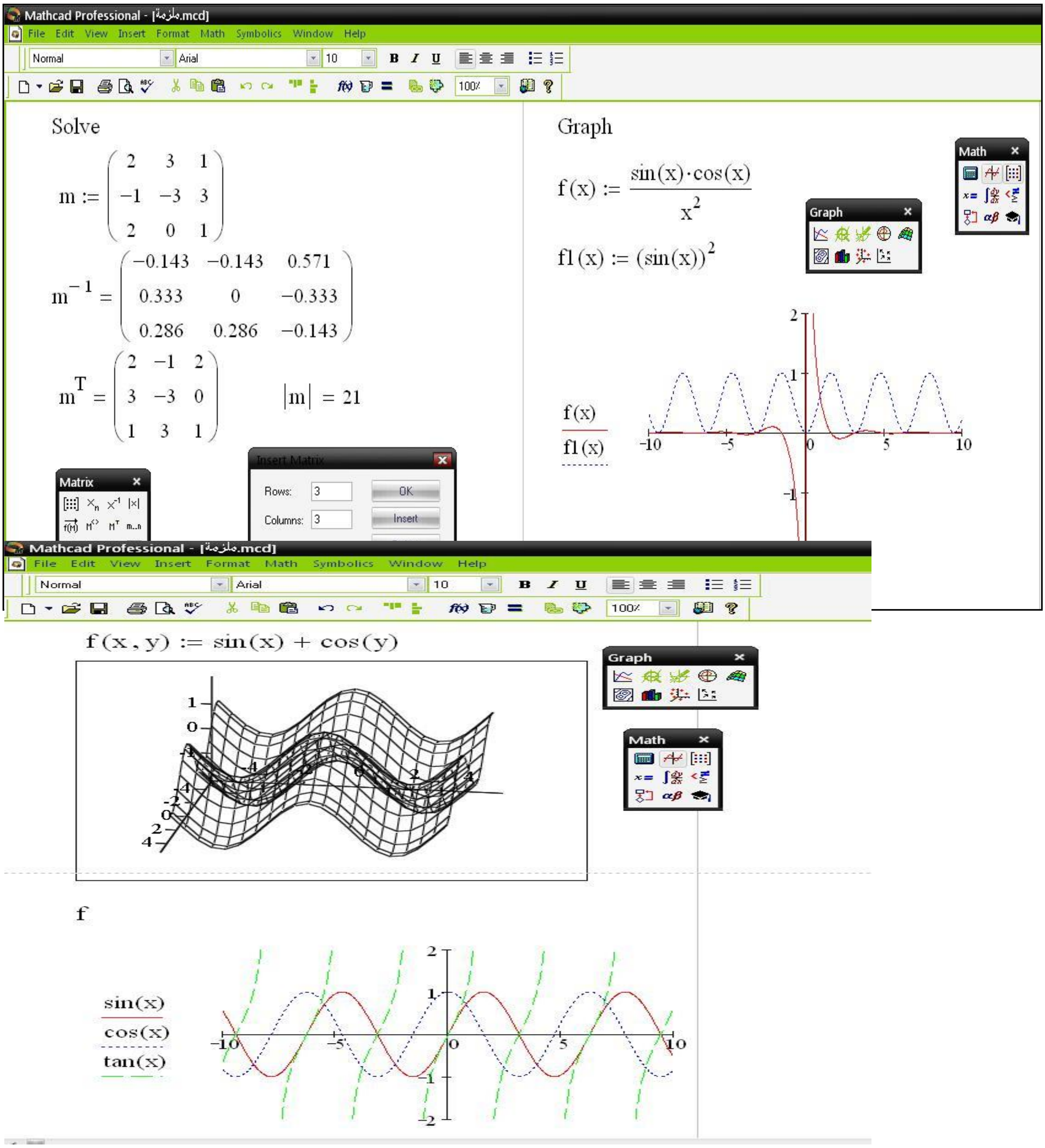
E-mail aljelawy2000@yahoo.com

$$\int (x^3 + 2x + 1)\, dx \rightarrow \frac{1}{4}\cdot x^4 + x^2 + x$$

$$\int \tan(2x)\, dx \rightarrow \frac{1}{4}\cdot \ln\left(2 + 2\cdot\tan(2\cdot x)^2\right)$$

$$\int \sin(2x) + \cos(3x)\, dx \rightarrow \frac{-1}{2}\cdot\cos(2\cdot x) + \frac{1}{3}\cdot\sin(3\cdot x)$$

$$\int \frac{1}{(1 + x^2)}\, dx \rightarrow \operatorname{atan}(x) \qquad \int e^{2x}\, dx \rightarrow \frac{1}{2}\cdot\exp(2\cdot x)$$



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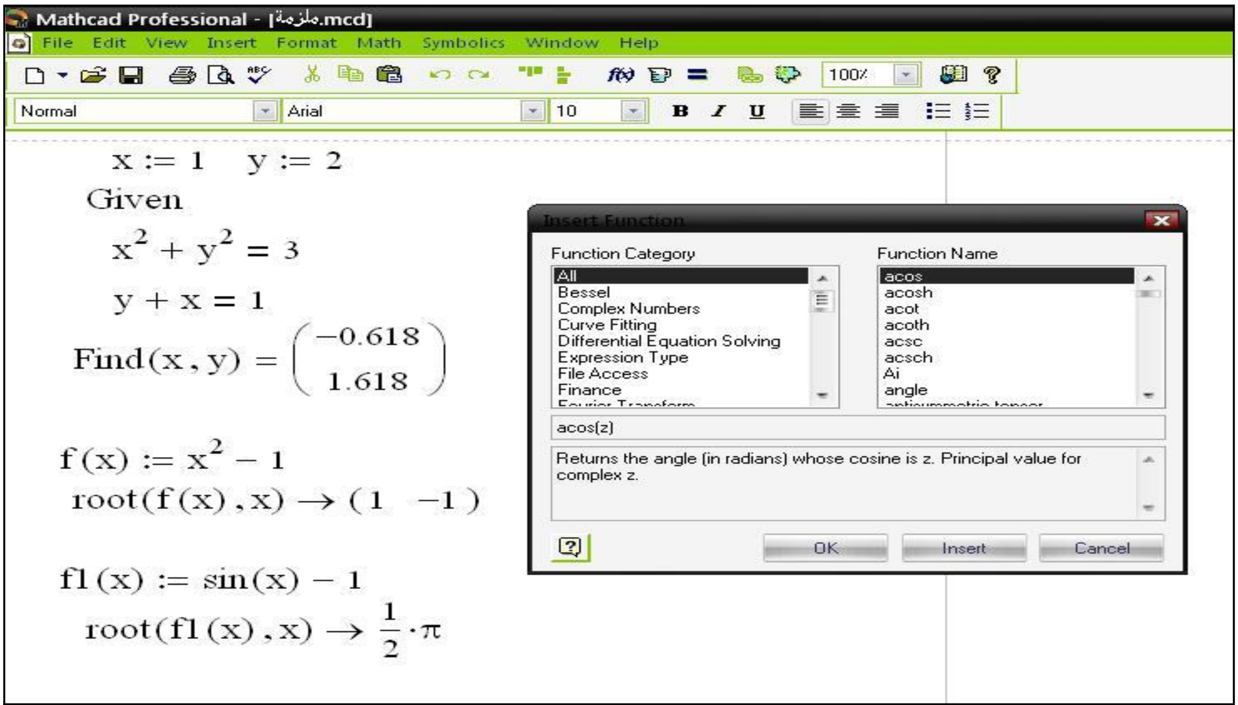

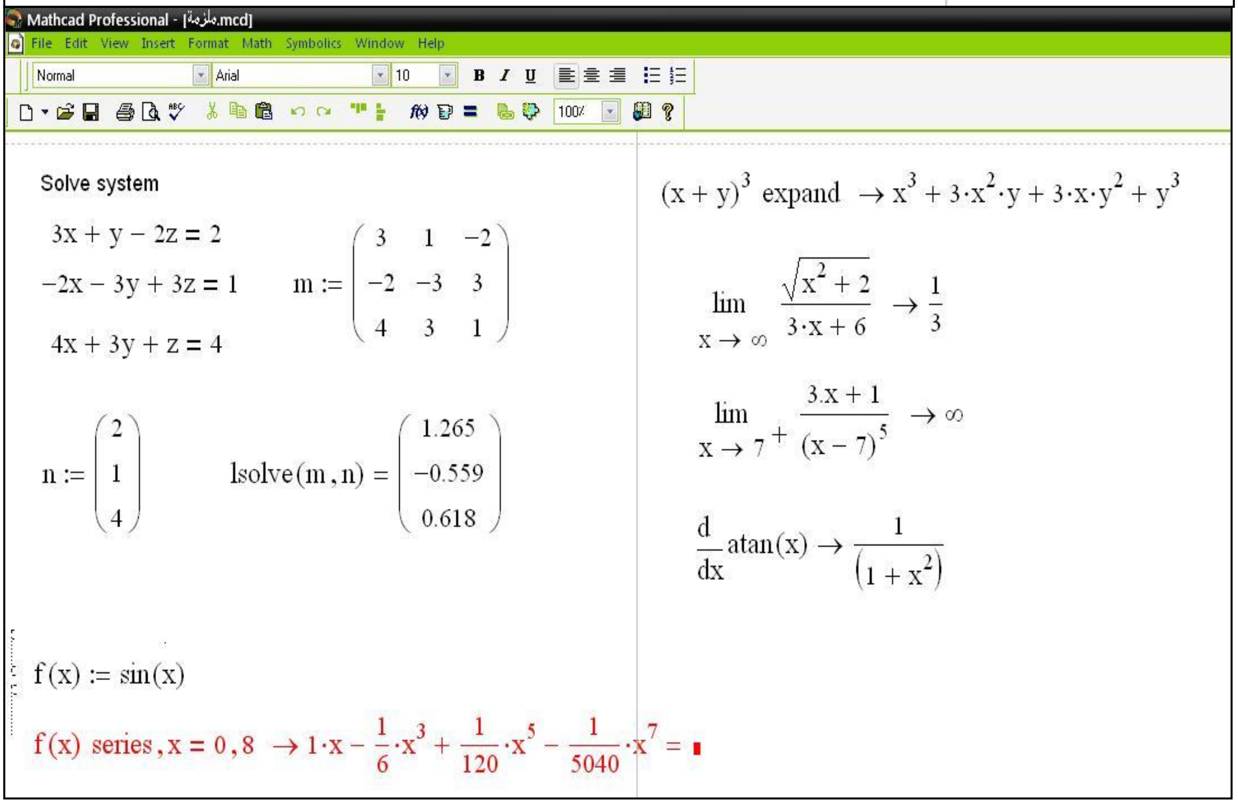

Solve Function

$$x^2 + 2x - 3 = 0 \text{ solve}, x \rightarrow \begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

$$x + 2 = 0 \text{ solve}, x \rightarrow -2$$

$$x^2 + 1 \text{ solve}, x \rightarrow \begin{pmatrix} i \\ -i \end{pmatrix}$$

$$x^2 + e^2 \text{ solve}, x \rightarrow \begin{pmatrix} i \cdot \exp(2)^{\frac{1}{2}} \\ -i \cdot \exp(2)^{\frac{1}{2}} \end{pmatrix}$$

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2- Series Function

$$f(x) := e^{x}$$

$$f(x)\ \text{series}, x = 1, 3 \rightarrow \exp(1) + \exp(1)\cdot(x-1) + \frac{1}{2}\cdot\exp(1)\cdot(x-1)^2$$

3- Expand , Factor, Simplify and Function

$$(a+b)^3\ \text{expand} \rightarrow a^3 + 3\cdot a^2\cdot b + 3\cdot a\cdot b^2 + b^3$$

$$a^3 + 3\cdot a^2\cdot b + 3\cdot a\cdot b^2 + b^3\ \text{factor} \rightarrow (a+b)^3$$

$$\frac{\frac{r+s}{s} + \frac{s}{r-s}}{\frac{s}{r-s}}\ \text{simplify} \rightarrow \frac{r^2}{s^2}$$

4- Float and Substitution Functions

$$2\cdot\operatorname{acos}(0)\ \text{float}, 7 \rightarrow 3.141593$$

$$e\ \text{float}, 40 \rightarrow 2.718281828459045235360287471352662497757$$

$$\frac{x+3}{x^2}\ \text{substitute}, x = (y+1)^2 \rightarrow \frac{\left[(y+1)^2 + 3\right]}{(y+1)^4}$$

$$\sin(\theta)^2 + \cos(\theta)^4\ \text{substitute}, \sin(\theta) = u, \cos(\theta) = k \rightarrow u^2 + k^4$$

$$a^2 + b^5 \left|\begin{array}{l}\text{substitute}, a = c+1, b = c-1\\ \text{factor}\end{array}\right. \rightarrow c\cdot\left(-9\cdot c + 7 + c^4 - 5\cdot c^3 + 10\cdot c^2\right)$$

E-mail *aljelawy2000@yahoo.com*

5- Collect Function

$$x^2 - a\cdot y^2\cdot x^2 + 2\cdot y^2\cdot x - x + x\cdot y \text{ collect}, x \rightarrow \left(-a\cdot y^2 + 1\right)\cdot x^2 + \left(y + 2\cdot y^2 - 1\right)\cdot x$$

$$x^2 - a\cdot y^2\cdot x^2 + 2\cdot y^2\cdot x - x + x\cdot y \text{ collect}, y \rightarrow \left(-a\cdot x^2 + 2\cdot x\right)\cdot y^2 + x\cdot y + x^2 - x$$

$$\frac{x+3}{x^2} \left| \begin{array}{l} \text{substitute}, x = (y+1)^2 \\ \text{expand} \end{array} \right. \rightarrow \frac{\left(y^2 + 2\cdot y + 4\right)}{\left(y^4 + 4\cdot y^3 + 6\cdot y^2 + 4\cdot y + 1\right)}$$

$$a^2 + b^5 \left| \begin{array}{l} \text{substitute}, a = c + 1, b = c - 1 \\ \text{factor} \end{array} \right. \rightarrow c\cdot\left(-9\cdot c + 7 + c^4 - 5\cdot c^3 + 10\cdot c^2\right)$$

$$\frac{3}{19} + \frac{47}{93} \text{ simplify} \rightarrow \frac{1172}{1767} \qquad \frac{3}{19.0} + \frac{47}{93} \text{ simplify} \rightarrow .66327108092812676854$$

$$\frac{x^2 - 3\cdot x - 4}{x - 4} + 2\cdot x - 5 \text{ simplify} \rightarrow 3\cdot x - 4 \qquad e^{2\cdot\ln(a)} \text{ simplify} \rightarrow a^2$$

$$\sin(x)^2 + \cos(x)^2 \text{ simplify} \rightarrow 1$$

$$30! \text{ simplify} \rightarrow 265252859812191058636308480000000$$

$$\frac{1}{x-1} + \frac{x}{x+3} - \frac{2\cdot x}{x+2} \text{ factor} \rightarrow \frac{-\left(2\cdot x^2 - 9\cdot x - 6 + x^3\right)}{(x-1)\cdot(x+3)\cdot(x+2)}$$

$$\left(\sum_{i=1}^{n} i^2\right) \text{yields} \rightarrow \left[\frac{1}{3}\cdot(n+1)^3 - \frac{1}{2}\cdot(n+1)^2 + \frac{1}{6}\cdot n + \frac{1}{6}\right]\cdot \text{yields}$$

$$\left[\prod_{k=2}^{10} \left(1 - \frac{1}{k^3}\right)\right] \text{yields} \rightarrow \frac{80926932541}{99532800000}\cdot \text{yields}$$

$$\int_{\pi}^{2\pi}\int_{0}^{\pi} \sin(x)\, dx\, dy \rightarrow 2\cdot\pi \qquad \int_{0}^{2}\int_{-2x}^{x^2} \frac{x^2+1}{x+1}\, dy\, dx \rightarrow \frac{32}{3} - 2\cdot\ln(3)$$

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