The effect of breakup on the total fusion reaction cross section of stable weakly bound nuclei

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Abstract:

In the present study, we have performed Coupled-Channel (CC) calculations to study the effect of coupling to the breakup channel on the calculations of the total reaction cross section $\pm_{fus}$ and the fusion barrier distribution at energies near and below the Coulomb barrier V_b for the systems $^{6}Li+^{209}Bi$, $^{7}Li+^{209}Bi$ and $^{9}Be+^{208}Pb$. The inclusion of breakup reaction enhances the calculations of the total reaction cross section in comparison with the recent available experimental data at energies near and below the Coulomb barrier. The inclusion of breakup channel is found to be very essential and modifies the calculations of the total fusion cross section markedly and describes the experimental data very well below and above the Coulomb barrier.

Keywords: Weakly bound nuclei, coupled channel, fusion cross section

تأثير التفكك على مقطع الأستطارة الكلي لتفاعل الأندماج للنوى المستقرة ضعيفة الترابط

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الخلاصة:

في هذه الدراسة **قمنا بأجراء حسابات أقتران القنوات** (Coupled-Channel) لدراسة تأثير **أقتران قناة التفكك على حسابات مقطع الأستطارة التام للتفاعل** $\pm_{fus}$ وتوزيع حاجز الأندماج عند الطاقات بالقرب وأسفل حاجز الجهد الكولومي V_b **للأنظمة** $^{6}Li+^{209}Bi$ ، $^{7}Li+^{209}Bi$ **و** $^{9}Be+^{208}Pb$. أن أعتماد تفاعل التفكك أدى الى تحسين حسابات مقطع الأستطارة التام للتفاعل مقارنة مع البيانات العملية المتوفرة حديثاً عند الطاقات بالقرب وأسفل حاجز الجهد **الكولومي**. لقد وجد بأن أعتماد تفاعل التفكك ضروري جداً وقاد الى تحسين الحسابات لمقطع الأستطارة التام للتفاعل بشكل جيد جداً أسفل وفوق حاجز الجهد الكولومي.

المفاتيح: النويات ضعيفة الترابط، **أقتران القنوات، مقطع أستطارة الأندماج**

1. Introduction

The study of nuclear reactions in collisions of weakly bound nuclei has attracted considerable interest in the last decades [P.R.S. Gomes *et al*., (2012); P.R.S. Gomes *et al*., (2011); R. Raabe, (2008); Canto *et al*., (2006); Bertulani et al., (2001); Hussein *et al*., (2003)]. In particular, several measurements of fusion and breakup cross sections in reactions induced by stable [P.R.S. Gomes *et al*., (2011)] and radioactive weakly bound nuclei have recently been made [M. Dasgupta *et al*., (1999); R. Raabe *et al*., (2004)]. These new data call for adequate theoretical tools for their interpretations.

The first estimates of the complete fusion cross section for weakly bound projectiles lead to conflicting results. While some calculations predicted a suppression of this cross section [Hussein *et al*., (1992)], others predicted its enhancement [C.H. Dasso, *et al*. (1994); Nunes, *et al*., (1999)]. In both cases, however, the calculations were quite schematic in their inclusion of the breakup channel. A more realistic coupled-channels calculations are performed [K. Hagino *et al*., (2000); Diaz-Torres and Thompson, (2002); Diaz-Torres *et al*., (2003)]. These calculations employed the Continuum Discretized Coupled-Channel (CDCC) method, which, although being the proper way to describe coupled-channels problems involving the continuum, makes the calculations more complicated. The aim of the present work is to perform Coupled-Channel calculations (CC) to study the effect of taking coupling of the breakup channel on the calculation of the total fusion reaction cross section and the fusion barrier distribution at energies near and below and Coloumb barrier V_b .

2. Coupled-channel formalism

The nuclear structure effects can be taken into account in a more quantal way using the coupled-channels method. In order to formulate the coupled-channels method, consider a collision between two nuclei in the presence of the coupling of the relative motion, $r = (r, \hat{r})$, to a nuclear intrinsic motion ξ . We assume the following Hamiltonian for this system [K. Hagino *et al*., (2012)],

$$H(r,\xi) = -\frac{\hbar^2}{2\mu}\nabla^2 + V(r) + H_0(\xi) + V_{coup}(r,\xi) \qquad (1)$$

where $H_0(\xi)$ and $V_{coup}(r,\xi)$ are the intrinsic and the coupling Hamiltonians, respectively. *V*(r) is the standard Woods-Saxon potential which has the form,

$$V(r) = \frac{-V_0}{1 + \exp\left[(r - r_0)/a\right]} \qquad (2)$$

where *a*, is the diffuseness parameter.

In general the intrinsic degree of freedom ξ has a finite spin. We therefore expand the coupling Hamiltonian in multipoles as [K. Hagino *et al*., (2012)],

$$V_{coup}(r,\xi) = \sum_{\lambda>0} f_\lambda(r) Y_\lambda(\hat{r}) \cdot T_\lambda(\xi) \qquad (3)$$

Here $Y_\lambda(\hat{r})$ are the spherical harmonics and $T_\lambda(\xi)$ are spherical tensors constructed from the intrinsic coordinate. The dot indicates a scalar product. The sum is taken over all values of λ except for λ = 0, which is already included in the bare potential, *V*(*r*).

For a fixed total angular momentum J and its z-component M, the expansion basis for the wavefunction in Eq. (2) are defined as [F. Muhammad, (2008)],

$$\langle \hat{r},\xi | (\alpha l I)\, JM \rangle = \sum_{m_l, m_I} \langle l m_l\, I\, m_I | JM \rangle Y_{l m_l}(\hat{r})\, \varphi_{\alpha I m_I}(\xi) \tag{4}$$

where l and I are the orbital and the intrinsic angular momenta, respectively. $\varphi_{\alpha I m_I}(\xi)$ are the wave functions of the intrinsic motion which obey,

$$H_0(\xi)\, \varphi_{\alpha I m_I}(\xi) = \varepsilon_{\alpha I}\, \varphi_{\alpha I m_I}(\xi) \tag{5}$$

Here, α denotes any quantum number besides the angular momentum. Expanding the total wave function with the channel wave functions as [K. Hagino *et al.*, (2012)],

$$\Psi_J(r,\xi) = \sum_{\alpha, l, I} \frac{u^J_{\alpha l I}(r)}{r} \langle \hat{r},\xi | (\alpha l I)\, JM \rangle \tag{6}$$

the coupled-channels equations for $u^J_{\alpha l I}(r)$ read [K. Hagino *et al.*, (2012)],

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{l(l+1)\hbar^2}{2\mu r^2} + V(r) - E + \varepsilon_{\alpha I} \right] u^J_{\alpha l I}(r) + \sum_{\alpha', l', I'} V^J_{\alpha l I;\alpha' l' I'}(r)\, u^J_{\alpha' l' I'}(r) = 0 \tag{7}$$

where the coupling matrix elements $V^J_{\alpha l I;\alpha' l' I'}(r)$ are given as [Edmonds, (1966)],

$$\begin{aligned} V^J_{\alpha l I;\alpha' l' I'}(r) &= \langle (\alpha l I) JM | V_{coup}(r,\xi) | (\alpha' l' I') JM \rangle, \\ &= \sum_{\lambda} (-1)^{I-I'+l'+J}\, f_\lambda(r) \langle l \| Y_\lambda \| l' \rangle \langle \alpha I \| T_\lambda \| \alpha' I' \rangle \\ &\quad \times \sqrt{(2l+1)(2I+1)} \begin{Bmatrix} I' & l' & J \\ l & I & \lambda \end{Bmatrix}. \end{aligned} \tag{8}$$

Notice that these matrix elements are independent of M. For the sake of simplicity of the notation, in the following let us introduce a simplified notation, n = {α, l, I}, and suppress the index J. The coupled-channels equation (6) then reads [K. Hagino *et al.*, (2012)],

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{l(l+1)\hbar^2}{2\mu r^2} + V(r) - E + \varepsilon_n \right] u_n(r) + \sum_{\alpha', l', I'} V_{nn'}(r)\, u_{n'}(r) = 0 \tag{9}$$

These coupled-channels equations are solved with the incoming wave boundary conditions of [K. Hagino *et al.*, (2012)],

$$\begin{aligned} u_n(r) &\approx \sqrt{\frac{k_{n_i}}{k_n(r)}}\, \mathfrak{T}^J_{nn_i} \exp\left(-i \int_{r_{abs}}^{r} k_n(r')\, dr' \right) & r \le r_{abs}, \\ &= H^{(-)}_{l n}(k_n r)\, \delta_{n,n_i} - \sqrt{\frac{k_{n_i}}{k_n}}\, S^J_{nn_i}\, H^{(+)}_{l n}(k_n r) & r \to \infty \end{aligned} \tag{10}$$

where n_i denotes the entrance channel. The local wave number $k_n(r)$ is defined by,

$$k_n(r) = \sqrt{\frac{2\mu}{\hbar^2}\left(E - \varepsilon_n - \frac{l_n(l_n+1)}{2\mu r^2} - V(r) \right)} \tag{11}$$

where $k_n = k_n(r=\infty) = \sqrt{2\mu\left(E-\varepsilon_n\right)/\hbar^2}$. Once the transmission coefficients $\mathfrak{T}^J_{nn_i}$ are obtained, the inclusive penetrability of the Coulomb potential barrier is given by,

$$P_J(E) = \sum_n \left| \mathfrak{T}^J_{nn_i} \right|^2 \tag{12}$$

The fusion cross section is then given by [K. Hagino *et al.*, (2012)],

$$\sigma_{fus}(E) = \frac{\pi}{k^2}\sum_J (2J+1)\, P_J(E) \tag{13}$$

The fusion barrier distribution is given by [L. F. Canto, *et al.*, 2006)],

$$D_f(E) = \frac{d^2(E\sigma_{fus})}{dE^2} = \pi R_b^2\left[-\frac{d}{dE}\left(\frac{1}{1+\exp\left[2\pi\,\frac{E-V_b}{\hbar\omega}\right]}\right)\right] \tag{14}$$

3. Results and Discussion

The coupled channeled calculations were performed using the code CCFULL [11]. This code solves the Schrödinger equation and the coupled equations exactly, making only the iso-centrifugal approximation.

The fusion cross sections are calculated using an incoming wave boundary condition. The nuclear potential was taken to be of a Woods-Saxon form. The depth V_0 and radius parameter r_0 used for the single barrier penetration calculations for the ^{6}Li+^{209}Bi, ^{7}Li+^{209}Bi, and ^{9}Be+^{208}Pb systems, were V_0=107 MeV, r_0=1.12 fm, V_0=113 MeV, r_0=1.12 fm, and V_0=198.00 MeV, r_0=1.10 fm, respectively.

The values of V_0 and r_0 were chosen such that the centroids of the calculated fusion barrier distributions for each system matched those measured. Also with these values of V_0 the CCFULL calculations could be carried out successfully at all measured $E_{c.m.}$. Choosing a small value of V_0 causes the potential pocket to disappears at larger values of angular momenta and fusion can no longer be defined [M. Dasgupta *et al.*, (2004)] in CCFULL.

The diffuseness parameter a of the Woods-Saxon nuclear potential was initially set to 0.63 fm for all three reactions. This value is very close to the predictions using the Woods-Saxon parametrization [R. A. Broglia, *et al.*, (1981)] of the Akyüz-Winther potential [Akyüz and Winther, (1981)] which gives a=0.62 fm, 0.63 fm and 0.64 fm, respectively, for the ^{6}Li, ^{7}Li, and ^{9}Be induced reactions.

The lowest collective states of the target nuclei were included in the CCFULL calculations. For ^{209}Bi, the septuplet and decuplet of identified states [ENSDF, (2012)] associated with the 3^- and 5^- collective excitations, respectively, were each approximated [ENSDF, (2012)] by a single level with an energy equal to that of the centroid of each multiplet and a deformation length corresponding to that of the combined states [34].

These states and the double octupole phonon state were included in the CCFULL calculations. For ^{208}Pb, the collective 3^- and 5^- states and double octupole phonons states were included in the harmonic limit.

The rotational coupling were taken into account with β_2 deformations parameters 0.87 and 0.80 for ^{6}Li and ^{7}Li, respectively.

In the reaction with ^{9}Be, couplings to the $\frac{5}{2}^-$ and $\frac{7}{2}^-$ states in the $K^\pi = \frac{3}{2}^-$ ground state rotational band with a β_2 of 0.92 were included. The comparison between our theoretical prediction for the total fusion cross section for the three systems ^{6}Li+^{209}Bi, ^{7}Li+^{209}Bi and ^{9}Be+^{208}Pb with their corresponding experimental data are shown in Figs.(1-3) panel (a), where the dotted line represent our calculations with no coupling, means the projectile and the target are considered to be inert. The dashed line represent the coupled channel (CC) calculations by considering vibrational coupling for the projectile nuclei and the target were taken to be inert. The solid line are the CC normalized by factor 0.66, 0.74 and 0.70 for the three systems $^{6}Li+^{209}Bi$, $^{7}Li+^{209}Bi$ and $^{9}Be+^{208}Pb$, respectively. Figs.(1-3) panel (b), shows the comparison of the fusion barrier distribution calculation with the measured values extracted from the experimental data. The comparison shows that with the previously mentioned scaled factors the results are quite well for the calculation of the fusion cross section and the fusion barrier distribution.

This scaling factor will be model dependent at the lowest energies, as the calculations are sensitive to the types of coupling and their strength. However, at energies around and above the average barrier, the calculation and, hence, the scaling factor is more robust, since changes in couplings or potential, within the constraints of the measured barrier distribution, do not change the suppression factor significantly.

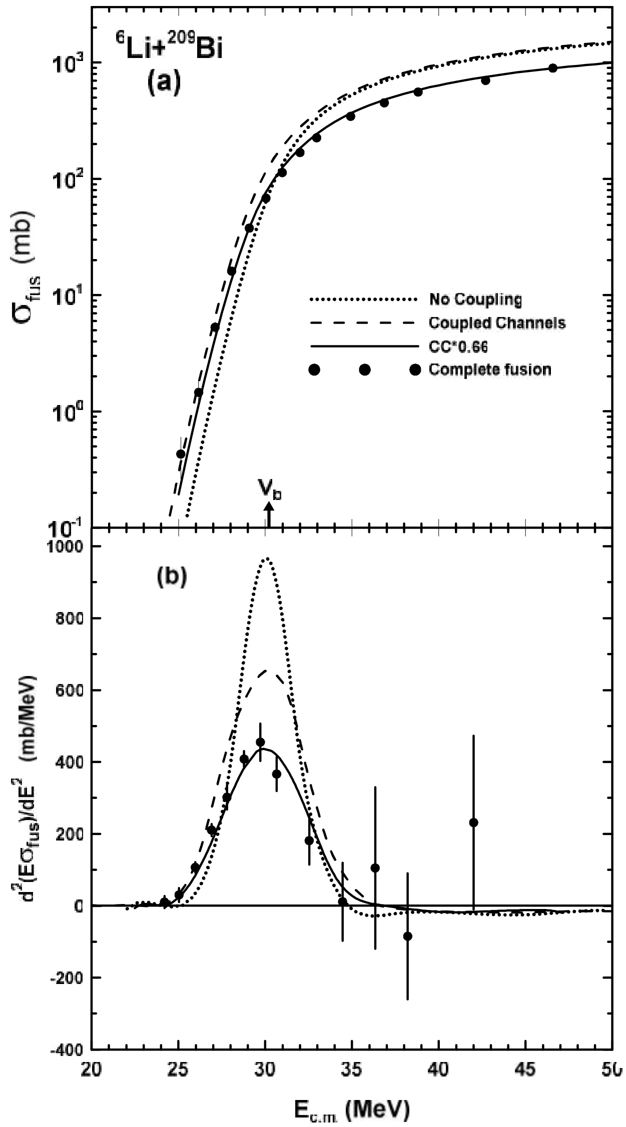

Fig. 1: The measured (filled circles) and calculated (a) complete fusion excitation function and (b) experimental barrier distribution for the fusion of $^{6}Li+^{209}Bi$. The dotted lines are the predictions of a single barrier penetration model (No coupling), and the dashed lines are the results of a coupled channels calculation (CC) .The full line is the latter calculation multiplied by 0.66 factor. The experimental data taken from Ref. [M. Dasgupta, *et al*., (2002)].

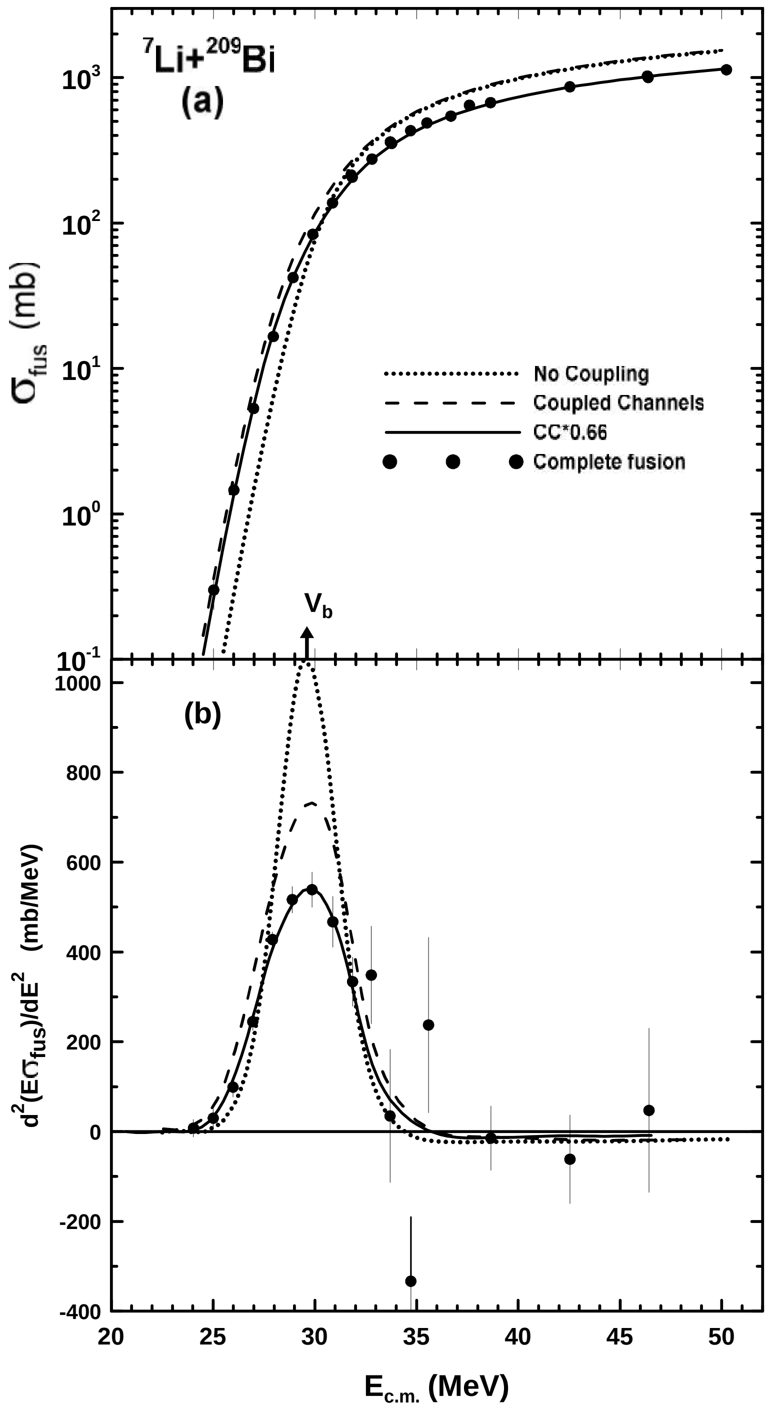

Fig. 2: The measured (filled circles) and calculated (a) complete fusion excitation function and (b) experimental barrier distribution for the fusion of ^{7}Li +^{209}Bi. The dotted lines are the predictions of a single barrier penetration model (No coupling), and the dashed lines are the results of a coupled channels calculation .The full line is the latter calculation multiplied by 0.74. The experimental data taken from Ref. [M. Dasgupta, *et al.*, (2002)].

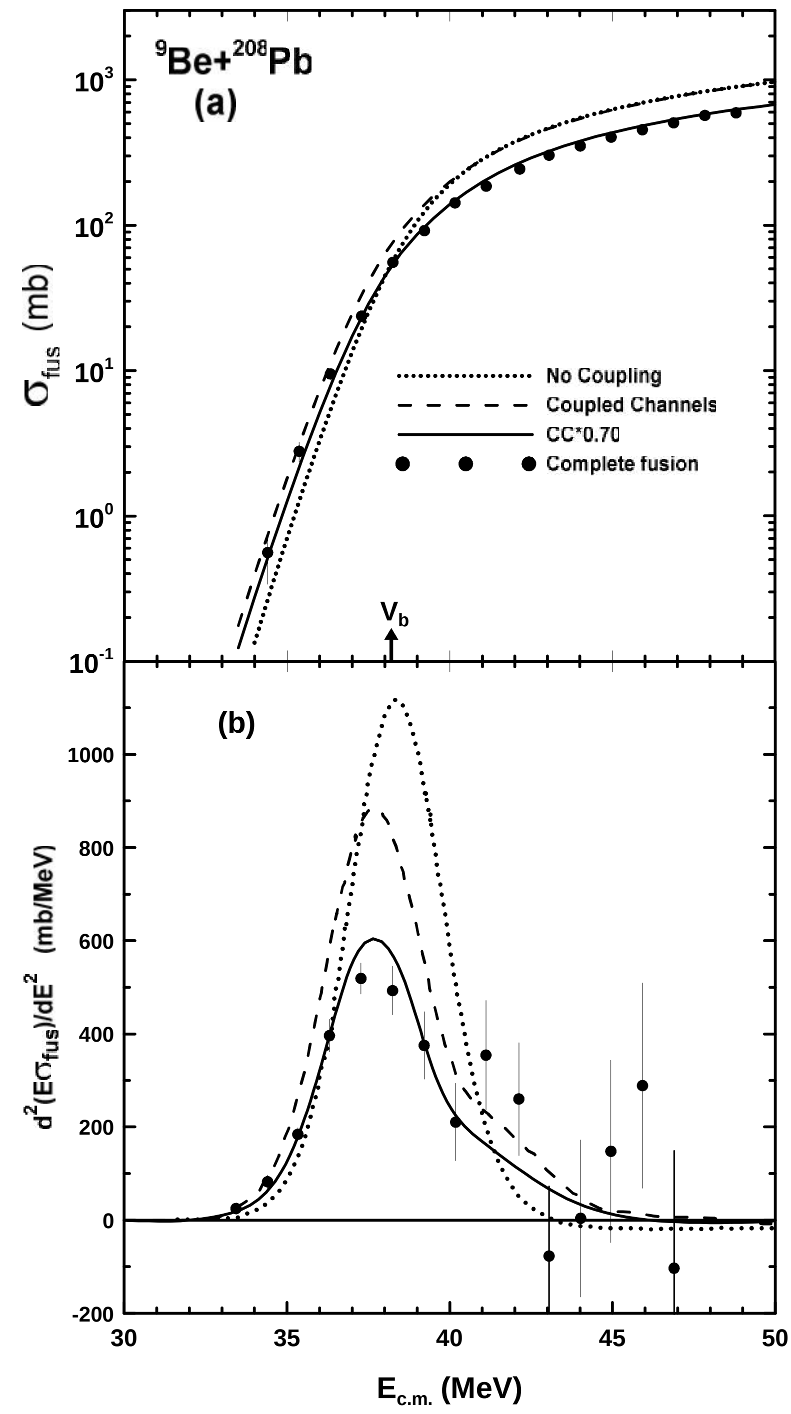

Fig. 3: The measured (filled circles) and calculated (a) complete fusion excitation function and (b) experimental barrier distribution for the fusion of $^{9}Be+^{208}Pb$. The dotted lines are the predictions of a single barrier penetration model (No coupling), and the dashed lines are the results of a coupled channels calculation .The full line is the latter calculation multiplied by the indicted factor. The experimental data taken from Ref. [M. Dasgupta, *et al*., (1999)].

4. Conclusions

At energies below the fusion barrier, there is a small enhancement in the cross sections, compared with the predictions of a single barrier model (no coupling), consistent with the low charge product of the reacting nuclei. However, at energies above the barrier the complete fusion cross sections are suppressed by ~30% compared with the expectations for fusion without breakup.

The results shows that the complete fusion cross sections at energies below the barrier will be enhanced due to couplings to bound and unbound (and transfer) states, but suppressed at energies above the barrier due to break up of the weakly bound light nucleus. However, thus far the models have either been qualitative, or have not attempted to separate complete fusion from incomplete fusion cross sections.

Coupling to the breakup channel is found to be very essential and it enhances the calculation of the fusion cross section and the fusion barrier distribution markedly below and above the barrier.

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