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S-open set in Bitopological space

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Abstract
The primary purpose of this paper is to introduce and study a new types of open sets called S-open sets , continuous , separation axioms are study with respect to the new open set .

الخلاصة
الهدف من هذا البحث هو دراسة نوع جديد من المجموعات المفتوحة في الفضاء الثنـائي التبولـوجي (X,P1,p2) تـسمى S- مفتوحة

Introduction
A triple (X,p_1,P_2) where X is anon empty set and P_1,P_2 are topologies on X , is called abitopological space and kelly [Kelly ,1963] initiated the study of such spaces. after that more mathematician working on the new space bitopological spaces deferent sets are defined and study in it .
if *A* be a subset of X, the interior (resp. closure) of *A* with respect to the topology *pi* (i = 1, 2) will be denoted by int $_{p}i$ (A) (resp. cl $_{p}i$ (A)).
The purpose of this paper is to define S-open sets and define separation axioms and continuous function associated S-open sets in bitopological spaces and investigate some of their properties and relations and its effects on some theorems and properties .

1-preminaries
in this section we define s-open set and we give some basic remark on it .
Definition(1-1):
Let (X,p1),(X,p2) are two topologicl on X asubset A of X is said to be S-openset in (x,p1,p2) iff A= int $p_1(U\cap V)$ such thatU,V are p_2-openset .the complement of S-open set is called S-closed set
Example(1-1)
Let X={a,b,c} ,p1={x , ø,{a},{c},{a,c}},p2={X, ø,{b },{a,c}
S.O(X)={X, ø,{a,c}**}**
Theorem(1-1):
The family of all S-open set is a topological space
Proof:

a) Ø and X is clearly that S-open set
b) Let G1,G2 two S-open sets
Then G1=int $p_1(U\cap V)$, G2=int$p_1(U1\cap V1)$,
Where U,V,U1,V1 are p2-open sets
then G1∩G2 = intp1(U∩V) ∩int p2(U1∩V1) = int p1(W∩H)
where W=U∩U1
H=V∩V1
Then G1∩G2 is S-open set

c)Let $\wedge$ be any index and $G\lambda$ is S-open set for every $\lambda\in\wedge$,then

$G\lambda=$ intp1($U\lambda \cap V\lambda$) fior some U_λ and V_λ be p_2-open sets then

$U\, G\lambda = U$ int p1($U\lambda\cap V\lambda$) =intp1($U(U\lambda\cap V\lambda)$)=intp1$[(U_{\lambda\in\wedge}(U\lambda)) \cap (U_{\lambda\in\wedge}(V\lambda))]$

Since $U_{\lambda\in\wedge}(U\lambda)$ and $(U_{\lambda\in\wedge}(V\lambda)$ be p2-open sets

Then $U_{\lambda\in\wedge}\, G\lambda$ is s-open set

Remark(1-1):

1/If Ais S-open set then it is P_1-open set .and the convers is not true

2/ If A is S-open set and B is P_1-open set thenA$\cap$B and AUB are P_1-open set

2- S-separation axioms and continuous function

In this section we will define separation axioms with respect to s-open sets.

Definition(2-1):

Abitopological space is said to be S-T0-space iff for each two distinct points x,y in X there exist s-open sets U such that $x\in U$, $y\notin U$ or there exist S-open set $y\in W$, $x\notin W$.

Definition(2-2):

Abitopological space is said to be S-T_1-space iff for each two distinct points x,y in X there exist two s-open sets U and W such that $x\in U$, $y\notin U$, $y\in W$, $x\notin W$.

Theorem(2-1):

let (X, P_1, P_2) be a bitopological space then X is S- T_1-space iff {x} is S-closed set for each x∈X.

Proof: see [sharma ,1963]

Definition(2-3):

Abitopological space is said to be S-T_2-space iff for each two distinct points x,y in X there exist two s-open sets U and W such that $x\in U$, $y\in W$,and $U\cap W=\phi$

Definition(2-4):

Abitopological space is said to be S-regular-space iff for each points x in X ands S-closed set H such that x∈ H there exist two s-open sets U and W such that $x\in U$, $H\subset W$,$U\cap V=\phi$

Definition(2-5):

Abitopological space is said to be S-T_3-space iff the bitoplogical space is S-T_1 and S-regular

Definition(2-6):

Abitopological space is said to be S-normal-space iff for each two S-closed sets H and F , such that $H\cap F=\phi$ there exist two s-open sets U and W such that $F\in U$, $H\subset W$, and $U\cap W=\phi$

Definition(2-7):
Abitopological space is said to be S-T_4-space iff the bitoplogical space is S-T_1 and S-normal .

Theorem(2-2): let (X, P_1, P_2) be abitopological space
If (X, P_1, P_2) is S-T_i-spase, i=0,1,2 then(X, P_1) is T_i-space where ,i=0,1,2
Proof:
Exist by definition

- the following diagram explain that the relation between the above separation axioms

P_1- T_0 P_1-T1 P_1-T_2 P_1-T_3 P_1-T_4
⇑ ⇑ ⇑ ⇑ ⇑
S-T0← S-T1← ST2← ←← S-T_3 ←S-T4

Examp(2-1):
Let X={ a,b,c} and P_1=D , D is the discrete topology on X which is T_i-space for i=0,1,2.
P_2={X, ∅,{a},{b},{a,b}}, S.O(X)=P_2 then (X,P_1,P_2) is not S-T_i-space for i=0,1,2.

Example(2-2) : let X={a,b,c,d} and
P_1={X,∅,{a},{c},{b},{a,c},{a,b},{b,c,d},{c,d},{b,c},{a,b,c},{a,c,d}}
P_2={X,∅, {{a},{c},{b},{c,d},{a,b},{a,c},{b,c},{b,c,d},{a,c,d}}
S.O(X)={X,∅,{a},{b},{c},{a,c},{a,b},{b,c},{c,d},{b,c,d},{a,b,c},{a,c,d}}
Then clearly that (X,P_1,P_2) is S-T_2 but not S-T_3

Example(2-3) : let X={a,b,c,d} and P_1=D
P_2={X,∅, {a},{c},{d},{a,d},{c,d},{a,b},{a,c},{b,c},{a,b,d}{a,b,c},{a,c,d}}
S.O(X)={ X,∅,{c},{a},{d},{a,b},{a,c},{a,d},{c,d},{a,d,b},{a,b,c}{a,c,d}}

Then clearly that (X,P_1,P_2) is S-T_3 but not S-T_4

Example (2-4) : let X={a,b,c,} and P_1={X,∅,{a},{c},{a,c}},
P_2={X,∅, {{a},{c},{a,c},{b,c}}
S.O(X)={X,∅,{a},{c},{a,c}}
Then clearly that (X,P_1,P_2) is S-T_o but not S-T_1

3- S-continuous functions

Now we define S-continuous function with some theorems on it we study .

Definition(3-1):
A mapping $f:(X, P_1, P_2) \rightarrow (Y, P^*_1 , P^*_2)$ is said to be S-continuous iff the inverse image of each S-open set in Y is P_1-open set in X

Theorem(3-1):
If $f: (x, P_1) \rightarrow (Y, P^*_1)$ is continuous then $f:(X, P_1, P_2) \rightarrow (Y, P^*_1 , P^*_2)$ is S-continuous

Proof
Let A is S-open set in Y then A is P^*_1-open set and since f is continuous then $f^{-1}(A)$ is P_1-open set and there for f is S-continuous.
Example (3-1)
Let X={a,b,c} , P_1={X,∅,{a},{b,c}} ,P_2={X,∅,{a},{a,c}}
Y={1,2,3} ,P_1^*={Y,∅,{1},{2},{1,2}}
P_2^*={Y,∅,{2}}
Let g: (X, P_1, P_2) → (Y, P^*_1 , P^*_2) defined by g(a)=2 , g(b)=1 , g(c)=3
Then clearly that g is S-continuous but not P_1-P_1^*- continuous
Remark(3-1):
The composition of two S-continuous functions is not S-continuous functions since not every P_1-open set is S-open set

Definition(3-2):
Amapping f:(X, P_1, P_2) → (Y, P^*_1 , P^*_2) is said to be
1/ S-open map iff f(u) is S-open set in Y for each S-open set U in X
2/ S-homeomorphism map iff f is 1-1,onto, S-continuous and S-open map

Definition (3-3):
Amapping f:(X, P_1, P_2) → (Y, P^*_1 , P^*_2) is said to be s-inn map iff the inverse image of each S-open set in Y is S-open set in X.
Theorem (3-2): if f:(X, P_1, P_2) → (Y, P^*_1 , P^*_2) is S-inn map then f is S-continuous map:
Proof: let U is S-open set in Y then $f^{-1}(U)$ is S-open set in X and since every S-open set is p1-open set then $f^{-1}(U)$ is p1-open set then f is S-continuous .
Example (3-2)
Let X={a,b,c} , P_1={X,∅,{a},{c}, {a,c}} ,P_2={X,∅,{a},{a,c}}
Y={1,2,3} ,P_1^*=D , D is the discrete topology on X .
P_2^*={Y,∅,{2},{1},{1,2}}
Let g: (X, P_1, P_2) → (Y, P^*_1 , P^*_2) defined by g(a)=1 , g(b)=3 , g(c)=2
Then clearly that g is S-continuous but not S-inn map
Theorem(3-3): let (X, P_1, P_2) be a bitopological space if X is S-T_i where ,i=0,1,2 then X is a topological properties with respect to S-T_i
Proof:
Exist by definition
Remark(3-2)
let (X, P_1, P_2) be a bitopological space
1- S-T_i ,i=3,4 is not topological properties
2- if f:(X, P_1, P_2) → (Y, P^*_1 , P^*_2) is S-inn map then T_i-space ,i=3,4 is a topological property.

Definition(3-4) : let (X, P_1, P_2) be abitopological space and Y be asubset of X then T_Y is the collection given by
P_{iY}= {D∩Y : D is S–open set} then (Y,P_{1Y} ,P_{2Y}) is called subspace of (X, P_1, P_2) .

Theorem (3-4): let (X, P_1, P_2) be a bitopological space S-T_i , i=1,2,3 are hereditary property .
Proof : see [Sharama , 1963] only we replace open set by S-open set .

Example(3-3)
Let X={a,b,c} , P_1={X,ø, {a},{c},{a,b}} p2={X,∅} Y={1,2,3} , P^*_1= {Y, ø, {2},{3},{1,3}, {2,3}}
S.O(Y)={ Y, ø, {1},{3},{1,3}}
, P^*_2={Y, ø, {2},{1,3}}
f:X→Y defined by f(a)=3 ,f(b)=1 ,f(c)=2 then f is S-continuous and
P_1- P^*_1 continuous

Example (3-4)
For the obove example if we define f as follow
f(a)=2 ,f(b)=3 ,f(c)=1 then f is not S-continuous since f^{-1}({1,3})={b,c} which is not P_1-open set

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