Algebraic Properties of Dispersion Measures

1. If a constant is added to or subtracted from each value, the measure of dispersion remains unchanged. That is: If $y_i = x_i + a, \ \forall \ a \in \mathbb{R}$, then:

i. $R_y = R_x$ ii. $M.D_y = M.D_x$ iii. $S_y = S_x$ iv. $S_y^2 = S_x^2$

2. If each value is multiplied or divided by a constant, both the new mean deviation and the new standard deviation are multiplied or divided by that value, while the variance is multiplied or divided by the square of the value.

That is: If $y_i = ax_i, \ \forall \ a \in \mathbb{R}^+$, then:

i. $R_y = aR_x$ ii. $M.D_y = aM.D_x$ iii. $S_y = aS_x$ iv. $S_y^2 = a^2S_x^2$

3. Non-negativity: All measures of dispersion yield positive values.

Example 1: The hemoglobin levels (g/dL) of 6 patients are: x_i: 12, 13, 14, 15, 16, 17.

Calculate $M.D, S^2$ and S.

Solution: Let $y_i = x_i - 12$. Then y_i: 0, 1, 2, 3, 4, 5.

$$\bar{y} = \frac{\sum y_i}{n} = \frac{15}{6} = 2.5$$

y_i	0	1	2	3	4	5
$|y_i - \bar{y}|$	2.5	1.5	0.5	0.5	1.5	2.5
$(y_i - \bar{y})^2$	6.25	2.25	0.25	0.25	2.25	6.25

$$M.D_x = M.D_y = \frac{\sum |y_i - \bar{y}|}{n} = \frac{9}{6} = 1.5$$

$$S_x^2 = S_y^2 = \frac{\sum (y_i - \bar{y})^2}{n - 1} = \frac{17.5}{5} = 3.5 \quad \text{and} \quad S_x = S_y = \sqrt{3.5} = 1.87$$

Example 2: Complete the following table:

Data	Arithmetic mean	Range	$M.D$	S^2	S
x_i: 1, 3, 5, 6, 10					
y_i: 101, 103, 105, 106, 110					
z_i: 5, 15, 25, 30, 50					

The solution is in the lecture.

Coefficient of Variation **(معامل الاختلاف)**

The standard deviation can be misleading when comparing variability between datasets, especially if they have different units or means. That's where the coefficient of variation comes in – it expresses the standard deviation as a percentage of the mean, allowing for relative comparisons. The formula is:

$$C.V = \frac{S}{\bar{x}} \times 100\%$$

Since the mean and standard deviation share the same unit as the original data, the coefficient of variation is unit-less, it's a dimensionless quantity.

Example 3: Find $C.V$ of the sample in example 1.

Solution:

$$\bar{x} = \frac{\sum x_i}{n} = 14.5 \text{ and } S = 1.87$$

$$C.V = \frac{1.87}{14.5} \times 100\% = 12.896\%$$

Standard Error (الخطأ القياسي)

Standard error measures how well a sample represents its population, using standard deviation. It's the deviation of the sample mean from the true population mean, showing how accurate the sample is. A big standard error means the population is varied, so samples will give different results. The formula is:

$$SE = \frac{S}{\sqrt{n}}$$

Example 4: Find SE of the sample in example 1.

Solution:

$$SE = \frac{S}{\sqrt{n}} = \frac{1.87}{\sqrt{6}} = 0.76$$

Standard Score   (الدرجة المعيارية)

The z-score tells you how many standard deviations a value is from the mean. It's calculated by subtracting the mean from the value and dividing by the standard deviation.

$$z = \frac{x_i - \overline{x}}{S}$$

Example 5: Suppose that the grades of the fourth-year students in the Medical Physics Department in the subjects of Health Physics and Biostatistics gave the following results:

Subjects	Health Physics	Biostatistics
Mean	55	75
Variance	225	144

If a student's score in Health Physics was 73 and in Biostatistics was 85, in which of the two subjects was the student's performance better?

Solution:

$z -$ score of Health Physics: $z = \dfrac{x_i - \overline{x}}{S} = \dfrac{73 - 55}{\sqrt{225}} = \dfrac{18}{15} = 1.2$

$z -$ score of Biostatistics: $z = \dfrac{x_i - \overline{x}}{S} = \dfrac{85 - 75}{\sqrt{144}} = \dfrac{10}{12} = 0.83$

Although the student's score in health physics was lower than his score in biostatistics, the student's performance was better in health physics.

H.W. In an analytical laboratory, the systolic blood pressure (mmHg) measurements for 50 patients were recorded as follows:

120, 145, 110, 130, 160, 125, 115, 140, 155, 135, 128, 148, 105, 138, 152, 118, 132, 142, 165, 122, 150, 112, 158, 129, 139, 149, 119, 136, 126, 144, 111, 134, 156, 121, 131, 141, 151, 109, 124, 137, 147, 117, 127, 143, 153, 107, 133, 123, 157, 113.

Calculate   1. $C.V$ 2. SE 3. z-score of 127.