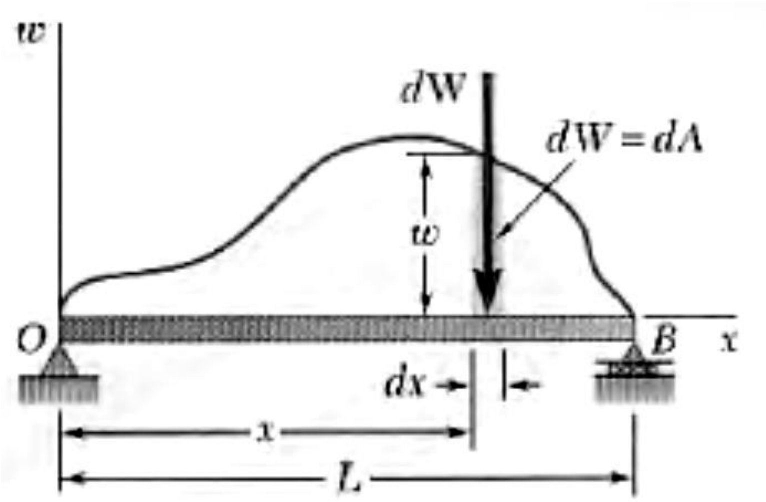
Distributed Loads on Beams:

The load on an element of the beam of length dx is :

$$dw = wdx$$

$$\int dw = \int wdx$$

$$W = \int_0^L wdx = area$$



∴ A distributed load on a beam may be replaced by a single concentrated load, the magnitude of which is equal to the area under the load curve and its line of action passes through the centroid of that area. This load may be used to determine reactions, but should not be used to compute internal forces and deformation.

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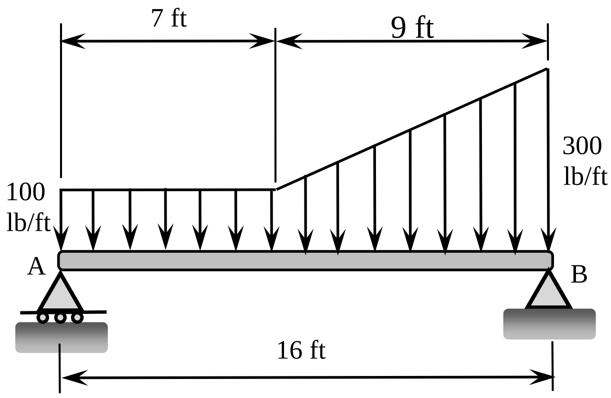
5.66

Determine the magnitude and location of the resultant of the distributed load shown. Also calculate the reactions at *A* and *B*.

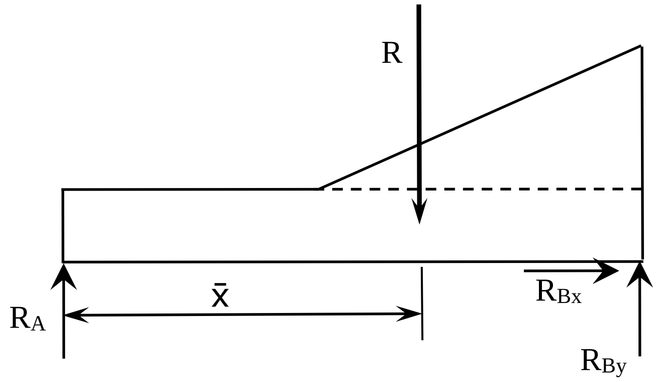
Comp.	A	$\bar{x}$	A $\bar{x}$
Rect.	=100*16 =1600	8	12800
Triangle	$=\frac{bh}{2}$ $=\frac{9*200}{2}$ $=900$	$=16-\frac{b}{3}$ $=16-\frac{9}{3}$ $=13$	11700
	Σ A = 2500		Σ A $\bar{x}$ = 24500



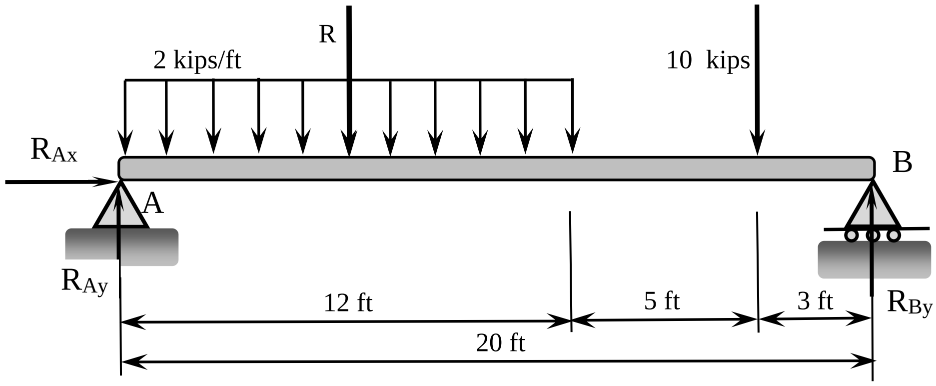
$$\overline{X} = \frac{\Sigma \bar{x} A}{A} = \frac{24500}{2500} = 9.8 in$$

$$R = A = 2500 lb$$

$$\Sigma M_B = 0$$
$$-R_A * 16 + 1600 * 8 + 900 * 3 = 0$$
$$R_A = \frac{1600*8+900*3}{16} = 968.75 lb$$
$$\Sigma F_x = 0 \Rightarrow R_{B_X} = 0$$
$$\Sigma F_y = 0$$
$$-R + R_{B_Y} + R_A = 0$$
$$R_{B_Y} = 2500 - 968.75 = 1531.25 lb$$



5.68 Determine the reactions at the beam supports for the given loading conditions.

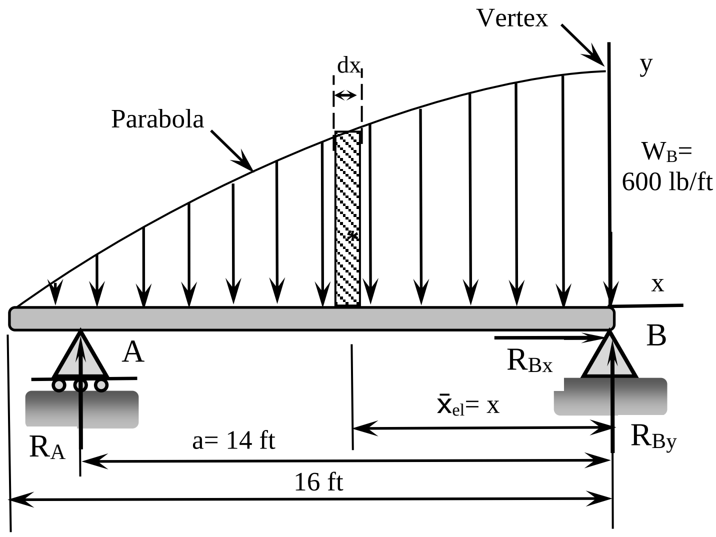


$$R = 12\,ft * 2 kips/\,ft$$
$$R = 24 kips$$
$$\bar{x} = 6\,ft$$
$$\Sigma M_A = 0$$
$$R_B * 20 - 10 * 17 - 24 * 6 = 0$$
$$R_B = \frac{10*17+24*6}{20}$$
$$R_B = 15.7 kips$$
$$\Sigma F_x = 0$$
$$\Rightarrow R_{A_x} = 0$$
$$\Sigma F_y = 0$$
$$R_{A_y} + R_B - 24 - 10 = 0$$
$$R_{A_y} = 18.3 kips$$

5.70 Determine the reactions at the beam supports for the given loading conditions.

$$(y-600)=-kx^2$$
$$at: x=0 \Rightarrow y=600$$
$$at: x=-16 \Rightarrow y=0$$
$$(0-600)=-k(16)^2$$
$$k=2.344$$

$$(y=-2.344x^2+600$$



$$A=\int_{-16}^{0} dA=\int_{-16}^{0} y dx=\int_{-16}^{0}(-2.344x^2+600)dx=(-2.344\frac{x^3}{3}+600x)\Big|_{-16}^{0}$$
$$=\left|0-(-2.344\frac{16^3}{3}+600*16\right|=3200-9600=6400$$
$$W=A=6400lb$$
$$\int_{-16}^{0}\overline{x_{el}}dA=\int_{-16}^{0} xy dx=\int_{-16}^{0} x(-2.344x^2+600)dx$$
$$=\int_{-16}^{0}(-2.344\frac{x^3}{3}+600x)dx=(-2.344\frac{x^4}{4}-\frac{600x^2}{2})\Big|_{-16}^{0}$$
$$=2.344\frac{(-16)^4}{4}-\frac{600(-16)^2}{2}=38404-76800=-38396$$
$$\overline{X}=\frac{\int\overline{x_{el}}dA}{A}=\frac{-38396}{6400}=-6\,ft$$
$$\Sigma M_B=0$$
$$-R_A*14+R*\overline{X}=0$$
$$R_A=\frac{6400*16}{14}=2742.857lb$$
$$\Sigma F_x=0 \Rightarrow R_{B_x}=0$$
$$\Sigma F_y=0$$
$$-R+R_{By}+R_A=0$$
$$R_{B_Y}=6400-2743=3657lb$$