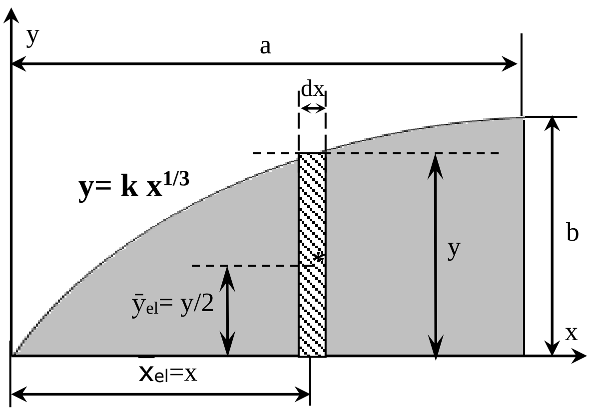
Determination of Centroids by Integration:

$\bar{x}$ **A = ∫ x dA** $\bar{y}$ **A = ∫ y dA**

$\bar{x}$ ***l*** **= ∫ x d*l*** $\bar{y}$ ***l*** **= ∫ y d*l***

5.34 Determine by direct integration the centroid of the area shown.



$$at: x = a; y = b \Rightarrow b = k(a)^{1/3}$$

$$\Rightarrow k = \frac{b}{a^{1/3}}$$

$$A = \int_0^a dA = \int_0^a y\,dx = \int_0^a kx^{1/3}dx$$

$$A = k\frac{x^{4/3}}{4/3}\Big|_0^a = \frac{3}{4}kx^{4/3}\Big|_0^a$$

$$A = \frac{3}{4}ka^{4/3}$$

$$\int_0^a \overline{x_{el}}\,dA = \int_0^a xy\,dx = \int_0^a x * kx^{1/3}dx = \int_0^a kx^{4/3}dx = k\frac{x^{7/3}}{7/3}\Big|_0^a$$

$$= \frac{3}{7}kx^{7/3}\Big|_0^a = \frac{3}{7}ka^{7/3}$$

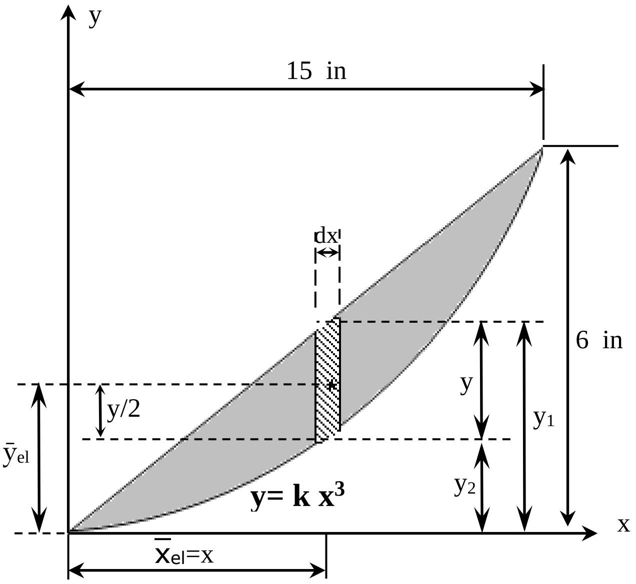
$$\overline{X} = \frac{\int \overline{x_{el}}\,dA}{A} = \frac{\frac{3}{7}ka^{7/3}}{\frac{3}{4}ka^{4/3}} = \frac{4}{7}a$$

$$\int_0^a \overline{y_{el}}\,dA = \int_0^a \frac{y}{2}y\,dx = \frac{1}{2}\int_0^a y^2dx = \frac{1}{2}\int_0^a (kx^{1/3})^2\,dx = \frac{1}{2}k^2\int_0^a x^{2/3}dx$$

$$= \frac{1}{2}k^2\frac{x^{5/3}}{5/3}\Big|_0^a = \frac{3}{10}k^2x^{5/3}\Big|_0^a = \frac{3}{10}k^2a^{5/3}$$

$$\overline{Y} = \frac{\int \overline{y_{el}}\,dA}{A} = \frac{\frac{3}{10}ka^{5/3}}{\frac{3}{4}ka^{4/3}} = \frac{2}{5}ka^{1/3} = \frac{2}{5}b$$

Solving **5.12** by integration : Determine by direct integration the centroid of the area shown.



line equation :

$$slop = \frac{\Delta y}{\Delta x} = \frac{6-0}{15-0} = \frac{2}{5}$$

$$y_1 = (\frac{\Delta y}{\Delta x})x \Rightarrow y_1 = \frac{2}{5}x$$

To fined the constant **k**:

at x=15 ; y=6

$$6 = k(15)^3 \Rightarrow k = \frac{6}{(15)^3}$$

$$k = 0.001777$$

$$A = \int_0^{15} dA = \int_0^{15} y dx = \int_0^{15} (y_1 - y_2)dx = \int_0^{15} (\frac{2}{5}x - 0.001777x^3)dx$$

$$A = (\frac{2}{5}\frac{x^2}{2} - 0.001777\frac{x^4}{4})\Big|_0^{15} = \frac{15^2}{5} - 0.001777\frac{15^4}{4}\Big|$$

$$A = 22.5$$

$$\int_0^{15} \overline{x_{el}} dA = \int_0^{15} xy dx = \int_0^{15} x(y_1 - y_2)dx = \int_0^{15} x(\frac{2}{5}x - 0.001777x^3)dx$$

$$= \int_0^{15} (\frac{2}{5}x^2 - 0.001777x^4)dx = (\frac{2}{5}\frac{x^3}{3} - 0.001777\frac{x^5}{5})\Big|_0^{15}$$

$$= \frac{2}{5} * \frac{15^3}{3} - 0.001777 * \frac{15^5}{5}$$

$$\int_0^{15} \overline{x_{el}} dA = 180$$

$$\overline{X} = \frac{\int \overline{x_{el}} dA}{A} = \frac{180}{22.5} = 8in$$

$$\overline{y_{el}} = \frac{y}{2} + y_2 = \frac{y_1 - y_2}{2} + y_2 = \frac{1}{2}y_1 - \frac{1}{2}y_2 + y_2 = \frac{1}{2}(y_1 + y_2)$$

$$\int_0^{15} \overline{y_{el}}\, dA = \int_0^{15} \frac{1}{2}(y_1 + y_2)y\,dx = \frac{1}{2}\int_0^{15} (y_1 y + y_2 y)dx$$

$$= \frac{1}{2}\int_0^{15} (y_1(y_1 - y_2) + y_2(y_1 - y_2))dx = \frac{1}{2}\int_0^{15} (y_1^2 - y_1 y_2 + y_1 y_2 - y_2^2)dx$$

$$= \frac{1}{2}\int_0^{15} (y_1^2 - y_2^2)dx = \frac{1}{2}\int_0^{15} ((\frac{2}{5}x)^2 - (0.001777x^3)^2)dx$$

$$= \frac{1}{2}\int_0^{15} (\frac{4}{25}x^2 - (0.001777)^2 x^6)dx$$

$$= \frac{1}{2}(\frac{4}{25} * \frac{x^3}{3} - (0.001777)^2 * \frac{x^7}{7}\Big|_0^{15}$$

$$= \frac{1}{2}(\frac{4}{75}(15)^3 - (0.001777)^2 * \frac{15^7}{7})$$

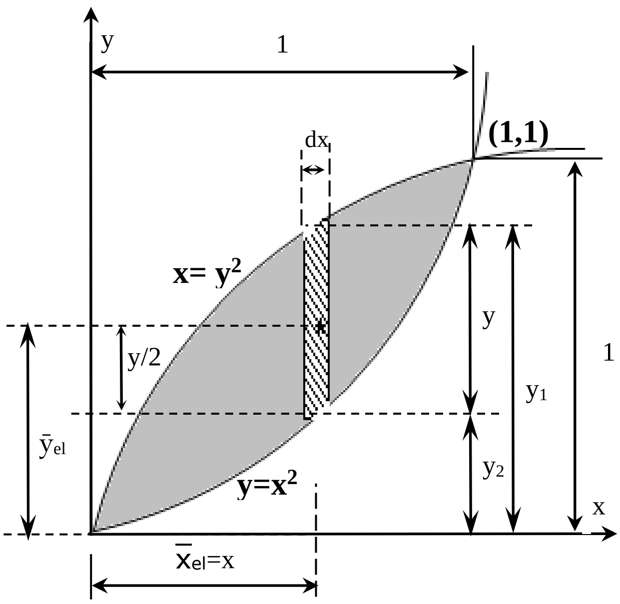
$$= \frac{1}{2}(180 - 77.136107)$$

$$\int_0^{15} \overline{y_{el}}\, dA = 51.43$$

$$\overline{Y} = \frac{\int \overline{y_{el}}\, dA}{A} = \frac{51.43}{22.5} = 2.285777in$$

5.42

Determine by direct integration the centroid of the area bounded by the curves **$y=x^2$** and **$x=y^2$.**



$$x = y_1^2 \Rightarrow y_1 = x^{1/2}; y_2 = x^2$$

$$y_1 = y_2 \Rightarrow x^2 = x^{1/2}; x^{3/2} = 1$$

$$\Rightarrow x = 1 \,\&\, y = 1$$

$$y = y_1 - y_2 = x^{1/2} - x^2$$

$$\overline{y_{el}} = \frac{y}{2} + y_2 = \frac{y_1 - y_2}{2} + y_2$$

$$= \frac{1}{2}y_1 - \frac{1}{2}y_2 + y_2$$

$$\overline{y_{el}} = \frac{1}{2}(y_1 + y_2)$$

$$A = \int_0^1 dA = \int_0^1 y dx = \int_0^1 (y_1 - y_2) dx = \int_0^1 (x^{1/2} - x^2) dx$$

$$A = \left(\frac{x^{3/2}}{3/2} - \frac{x^3}{3}\right)\Bigg|_0^1 = \frac{2*(1)^{3/2}}{3} - \frac{1(1)^3}{3} = \frac{1}{3}$$

$$A = 0.333$$

$$\int_0^1 \overline{x_{el}} dA = \int_0^1 xy dx = \int_0^1 x(y_1 - y_2) dx = \int_0^1 x(x^{1/2} - x^2) dx$$

$$= \int_0^1 (x^{3/2} - x^3) dx = \left(\frac{x^{5/2}}{5/2} - \frac{x^4}{4}\right)\Bigg|_0^1 = \frac{2}{5} * (1)^{5/2} - \frac{1^4}{4}$$

$$\int_0^1 \overline{x_{el}} dA = 0.15$$

$$\overline{X} = \frac{\int \overline{x_{el}} dA}{A} = \frac{0.15}{0.333} = 0.45$$

$$\int_0^1 \overline{y_{el}} dA = \int_0^1 \frac{1}{2}(y_1 + y_2) y dx = \frac{1}{2}\int_0^1 (y_1 y + y_2 y) dx$$

$$= \frac{1}{2}\int_0^1 (y_1(y_1 - y_2) + y_2(y_1 - y_2)) dx = \frac{1}{2}\int_0^1 (y_1^2 - y_1 y_2 + y_1 y_2 - y_2^2) dx$$

$$= \frac{1}{2}\int_0^1 (y_1^2 - y_2^2) dx = \frac{1}{2}\int_0^1 ((x^{1/2})^2 - (x^2)^2) dx = \frac{1}{2}\int_0^1 (x - x^4) dx$$

$$= \frac{1}{2}\left(\frac{x^2}{2} - \frac{x^5}{5}\right)\Bigg|_0^1 = \frac{1}{2}\left(\frac{(1)^2}{2} - \frac{(1)^5}{5}\right) = \frac{1}{4} - \frac{1}{10}$$

$$\int_0^1 \overline{y_{el}} dA = 0.15$$

$$\overline{Y} = \frac{\int \overline{y_{el}} dA}{A} = \frac{0.15}{0.333} = 0.45$$