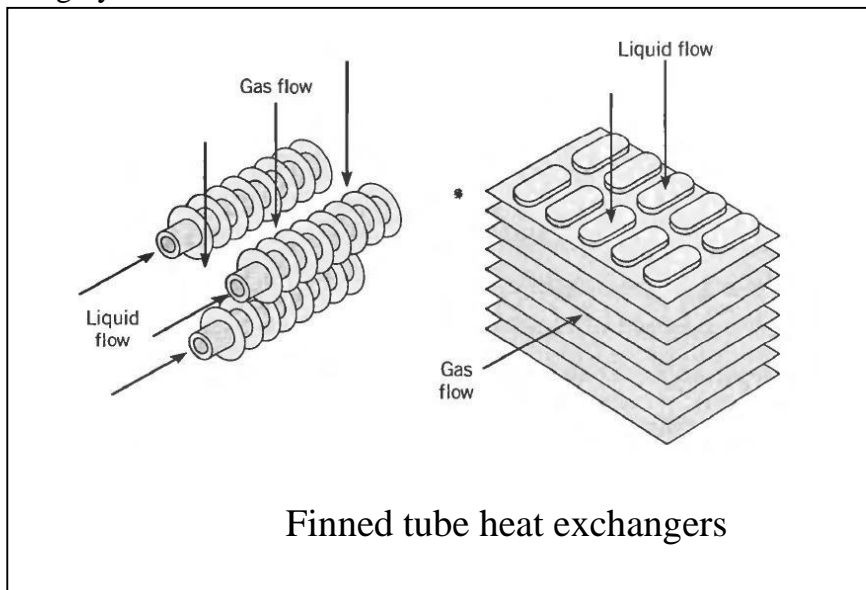


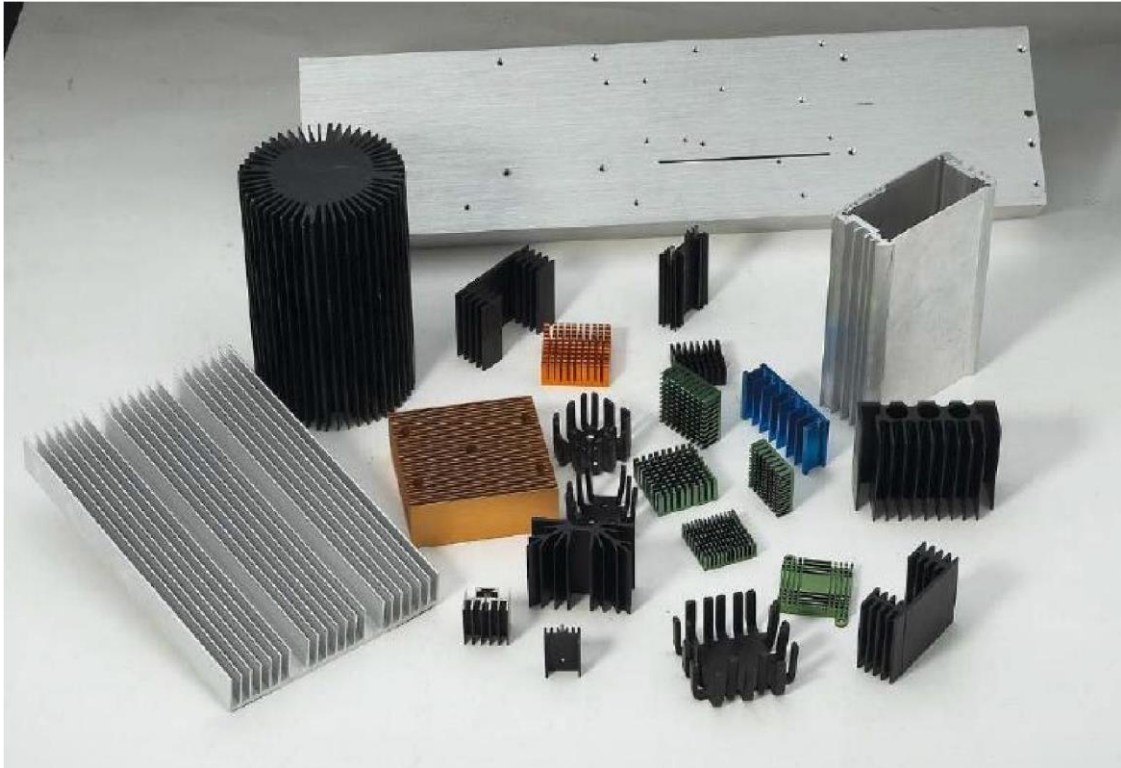
Introduction

$$Q_{\text{conv}} = hA_s (T_s - T_\infty)$$

There are two ways to increase the rate of heat transfer •
to increase the convection heat transfer coefficient (h)

- to increase the surface area A_s
- Increasing (h) may require the installation of a pump or fan,
- The alternative is to increase the surface area by attaching to the surface extended surfaces called fins
- made of highly conductive materials such as aluminum



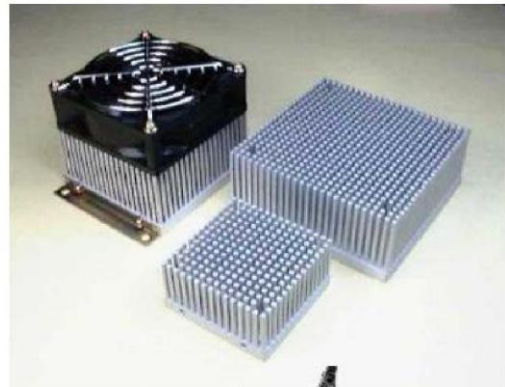


Heat sinks

Pin fins

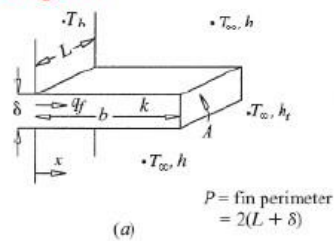


A splayed pin fin heat sink.



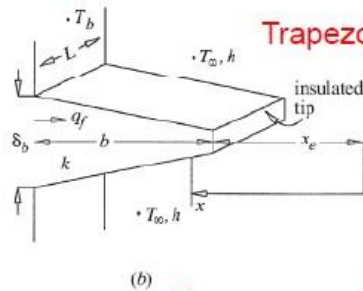
Longitudinal fins

Rectangular

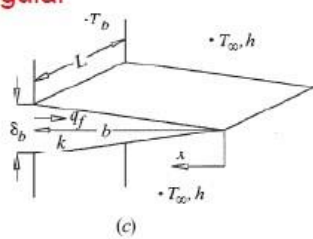


$P = \text{fin perimeter} = 2(L + \delta_b)$

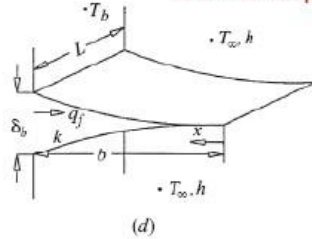
Trapezoidal



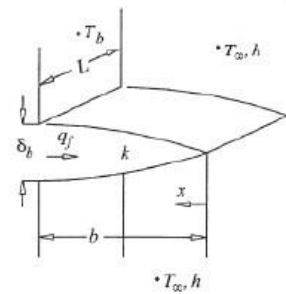
Triangular



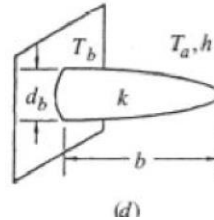
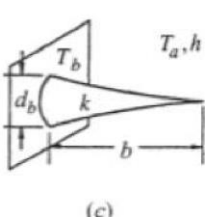
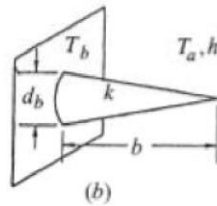
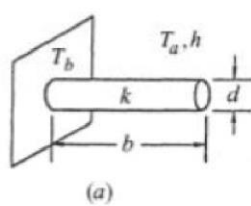
Concave parabolic



Convex parabolic

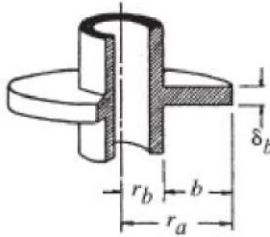


Pins:

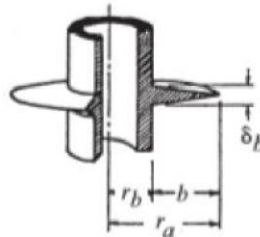


Radial fin coffee cup

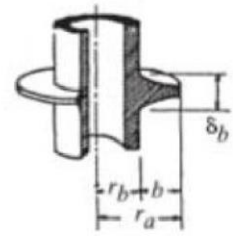
Radial fins:



Rectangular profile



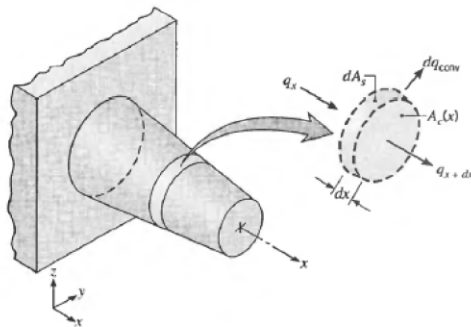
Triangular profile



Hyperbolic profile

Fin Equation

Fin



$$\dot{Q}_x = -kA_c \frac{dT}{dx}$$

$$\dot{Q}_{x+dx} = \dot{Q}_x + \frac{d\dot{Q}_x}{dx} dx$$

$$d\dot{Q}_{conv} = hdA_s(T - T_\infty)$$

Energy Balance:

$$\dot{Q}_x = \dot{Q}_{x+dx} + d\dot{Q}_{conv} = \dot{Q}_x + \frac{d\dot{Q}_x}{dx} dx + hdA_s(T - T_\infty)$$

$$\frac{d}{dx} \left(A_c \frac{dT}{dx} \right) - \frac{h}{k} \frac{dA_s}{dx} (T - T_\infty) = 0$$

$$\frac{d^2T}{dx^2} + \frac{1}{A_c} \frac{dA_c}{dx} \left(\frac{dT}{dx} \right) - \left(\frac{1}{A_c} \frac{h}{k} \frac{dA_s}{dx} \right) (T - T_\infty) = 0$$

Equation

Fins with uniform cross sectional area

$$\frac{d^2 T}{dx^2} + \frac{1}{A_c} \frac{dA_c}{dx} \left(\frac{dT}{dx} \right) - \left(\frac{1}{A_c} \frac{h}{k} \frac{dA_s}{dx} \right) (T - T_\infty) = 0$$

$$\frac{dA_c}{dx} = 0$$

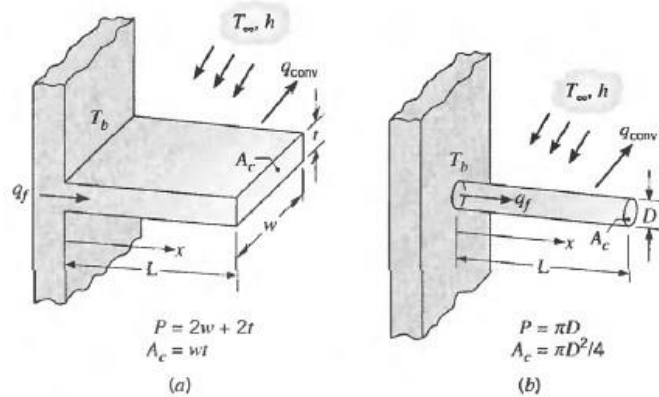
$$A_s = Px \quad \frac{dA_s}{dx} = P$$

$$\frac{d^2 T}{dx^2} - \left(\frac{hP}{kA_c} \right) (T - T_\infty) = 0$$

Excess temperature θ

$$\theta(x) \equiv T(x) - T_\infty$$

$$\frac{d^2 \theta}{dx^2} - m^2 \theta = 0$$



Straight fins of uniform cross section

(a) Rectangular Fin (b) Pin fin

General solution and Boundary Conditions

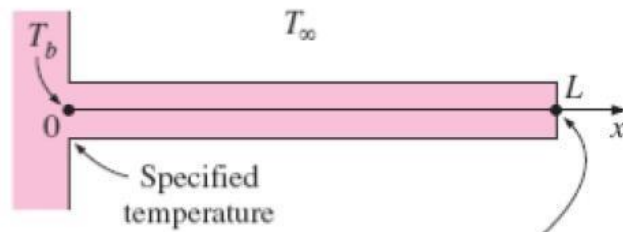
$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

$$m^2 = \frac{hP}{kA_c}$$

$$\theta(x) \equiv T(x) - T_\infty$$

The general solution is of the form

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$



- (a) Specified temperature
- (b) Negligible heat loss
- (c) Convection
- (d) Convection and radiation

Infinitely Long Fin ($T_{\text{fin tip}} = T_{\infty}$)

Case 1

Boundary condition at the fin tip: $\theta(L) = T(L) - T_{\infty} = 0$ as $L \rightarrow \infty$

The general solution is of the form

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

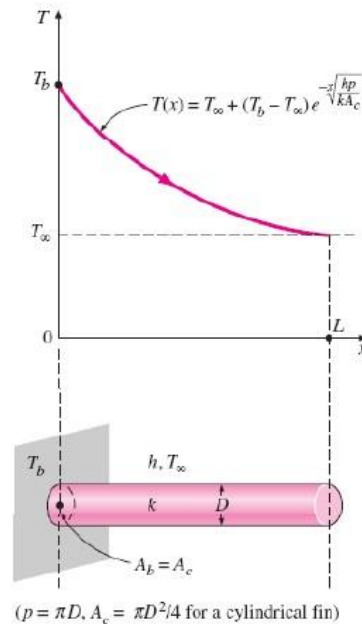
Possible when $C_1 \rightarrow 0$

$$\theta(x) = C_2 e^{-mx}$$

Apply boundary condition at base and find T

$$T(x) = T_{\infty} + (T_b - T_{\infty}) e^{-x \sqrt{\frac{hP}{kA_c}}}$$

$$\bullet \quad \dot{Q}_{\text{longfin}} = -kA_c \left. \frac{dT}{dx} \right|_{x=0} = \sqrt{hPkA_c} (T_b - T_{\infty})$$



Case 2

Insulated tip

General Sol: $\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$

$$\text{BC 1: } \theta(0) = T_b - T_{\infty} \equiv \theta_b$$

$$\text{BC 2: } \left. \frac{d\theta}{dx} \right|_{x=L} = 0$$

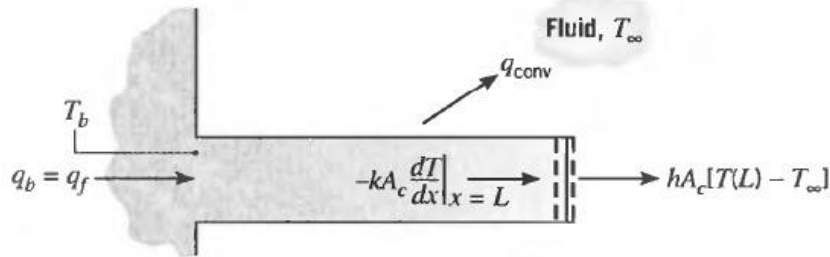
$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x)}{\cosh mL}$$

$$q_f = q_b = -kA_c \left. \frac{dT}{dx} \right|_{x=0} = -kA_c \left. \frac{d\theta}{dx} \right|_{x=0}$$

$$q_f = \sqrt{hPkA_c} \theta_b \tanh mL$$

Case 3

Convection from tip



q denotes \dot{Q}

BCs

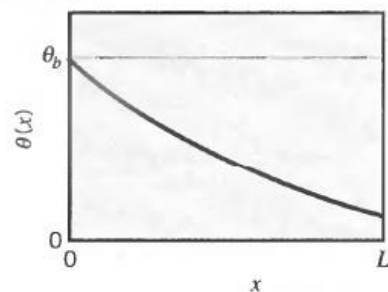
$$T(0) = T_\infty$$

$$\theta(0) = T_b - T_\infty \equiv \theta_b$$

$$hA_c [T(L) - T_\infty] = -kA_c \frac{dT}{dx} \Big|_{x=L}$$

$$h\theta(L) = -k \frac{d\theta}{dx} \Big|_{x=L}$$

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x) + (h/mk) \sinh m(L-x)}{\cosh mL + (h/mk) \sinh mL}$$



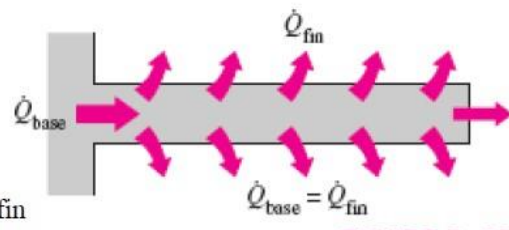
Heat transfer from the fin surface:

$$\dot{Q}_f = \dot{Q}_b = -kA_c \left. \frac{dT}{dx} \right|_{x=0} = -kA_c \left. \frac{d\theta}{dx} \right|_{x=0}$$

$$\dot{Q}_f = \sqrt{hPkA_c} \theta_b \frac{\sinh mL + (h/mk) \cosh mL}{\cosh mL + (h/mk) \sinh mL}$$

Another way of finding Q

$$\dot{Q}_{fin} = \int_{A_{fin}} h[T(x) - T_\infty] dA_{fin} = \int_{A_{fin}} h\theta(x) dA_{fin}$$



Under steady conditions, heat transfer from the exposed surfaces of the fin is equal to heat conduction to the fin at the base.

Prescribed temperature

Case 4

This is a condition when the temperature at the tip is known (for example, measured by a sensor)

اثبت ذلك ؟ (اشتقاق)

$$\frac{\theta}{\theta_b} = \frac{(\theta_L/\theta_b) \sinh mx + \sinh m(L-x)}{\sinh mL}$$

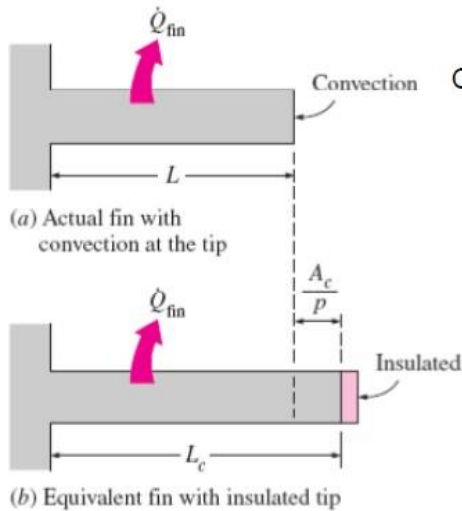
$$q_f = \sqrt{hPkA_c} \theta_b \frac{\cosh mL - (\theta_L/\theta_b)}{\sinh mL}$$

Temperature distribution and heat loss for fins of uniform cross section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q_f
A	Convection heat transfer: $h\theta(L) = -k d\theta/dx _{x=L}$	$\frac{\cosh m(L-x) + (h/mk) \sinh m(L-x)}{\cosh mL + (h/mk) \sinh mL}$	$M \frac{\sinh mL + (h/mk) \cosh mL}{\cosh mL + (h/mk) \sinh mL}$
B	Adiabatic $d\theta/dx _{x=L} = 0$	$\frac{\cosh m(L-x)}{\cosh mL}$	$M \tanh mL$
C	Prescribed temperature: $\theta(L) = \theta_L$	$\frac{(\theta_L/\theta_b) \sinh mx + \sinh m(L-x)}{\sinh mL}$	$M \frac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$
D	Infinite fin ($L \rightarrow \infty$): $\theta(L) = 0$	e^{-mx}	M

$\theta \equiv T - T_\infty$ $m^2 \equiv hP/kA_c$
 $\theta_b = \theta(0) = T_b - T_\infty$ $M = \sqrt{hPkA_c} \theta_b$

Corrected fin length



Corrected fin length:
$$L_c = L + \frac{A_c}{P}$$

Multiplying the relation above by the perimeter gives

$$A_{corrected} = A_{fin(lateral)} + A_{tip}$$

$$L_{c,rectangularfin} = L + \frac{t}{2}$$

$$L_{c,cylindricalfin} = L + \frac{D}{4}$$

Corrected fin length L_c is defined such that heat transfer from a fin of length L_c with insulated tip is equal to heat transfer from the actual fin of length L with convection at the fin tip.

Fin Efficiency

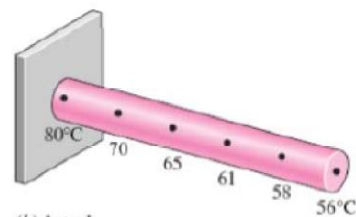
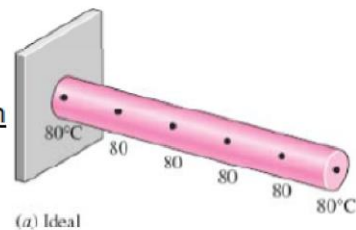
$$\eta_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{fin,max}} = \frac{\text{Actual heat transfer rate from the fin}}{\text{Ideal heat transfer rate from the fin if the entire fin were at base temperature}}$$

In the limiting case of zero thermal resistance or infinite thermal conductivity ($k \rightarrow \infty$), the temperature of the fin will be uniform at the base value of T_b .

$$\dot{Q}_{fin} = \eta_{fin} \dot{Q}_{fin,max} = \eta_{fin} h A_{fin} (T_b - T_\infty)$$

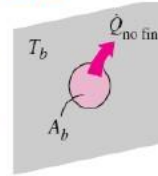
$$\eta_{longfin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{fin,max}} = \frac{\sqrt{hPkA_c} (T_b - T_\infty)}{hA_{fin} (T_b - T_\infty)} = \frac{1}{L} \sqrt{\frac{kA_c}{hp}} = \frac{1}{mL}$$

$$\eta_{insulatedtip} = \frac{\dot{Q}_{fin}}{\dot{Q}_{fin,max}} = \frac{\sqrt{hPkA_c} (T_b - T_\infty) \tanh mL}{hA_{fin} (T_b - T_\infty)} = \frac{\tanh mL}{mL}$$

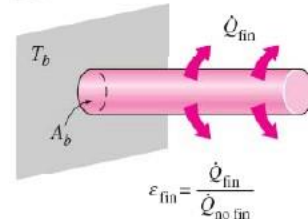


Fin Effectiveness

$$\varepsilon_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{nofin}} = \frac{\dot{Q}_{fin}}{hA_b(T_b - T_\infty)} = \frac{\text{Heat transfer rate from the fin of base area } A_b}{\text{Heat transfer rate from the surface of area } A_b}$$



$$\varepsilon_{longfin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{nofin}} = \frac{\sqrt{hPkA_c}(T_b - T_\infty)}{hA_b(T_b - T_\infty)} = \sqrt{\frac{kP}{hA_c}}$$



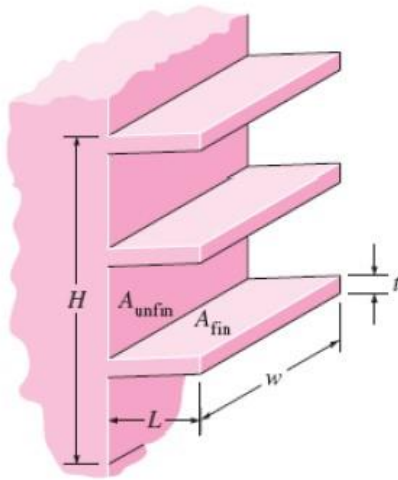
1. k should be as high as possible, (copper, aluminum, iron). Aluminum is preferred: low cost and weight, resistance to corrosion.
2. p/A_c should be as high as possible. (Thin plate fins and slender pin fins)
3. Most effective in applications where h is low. (Use of fins justified if when the medium is gas and heat transfer is by natural convection).

Fin Effectiveness

$$\varepsilon_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{nofin}} = \frac{\dot{Q}_{fin}}{hA_b(T_b - T_\infty)} = \frac{\text{Heat transfer rate from the fin of base area } A_b}{\text{Heat transfer rate from the surface of area } A_b}$$

- $\varepsilon_{fin} = 1$ Does not affect the heat transfer at all.
- $\varepsilon_{fin} < 1$ Fin act as insulation (if low k material is used)
- $\varepsilon_{fin} > 1$ Enhancing heat transfer (use of fins justified if $\varepsilon_{fin} > 2$)

Overall Fin Efficiency



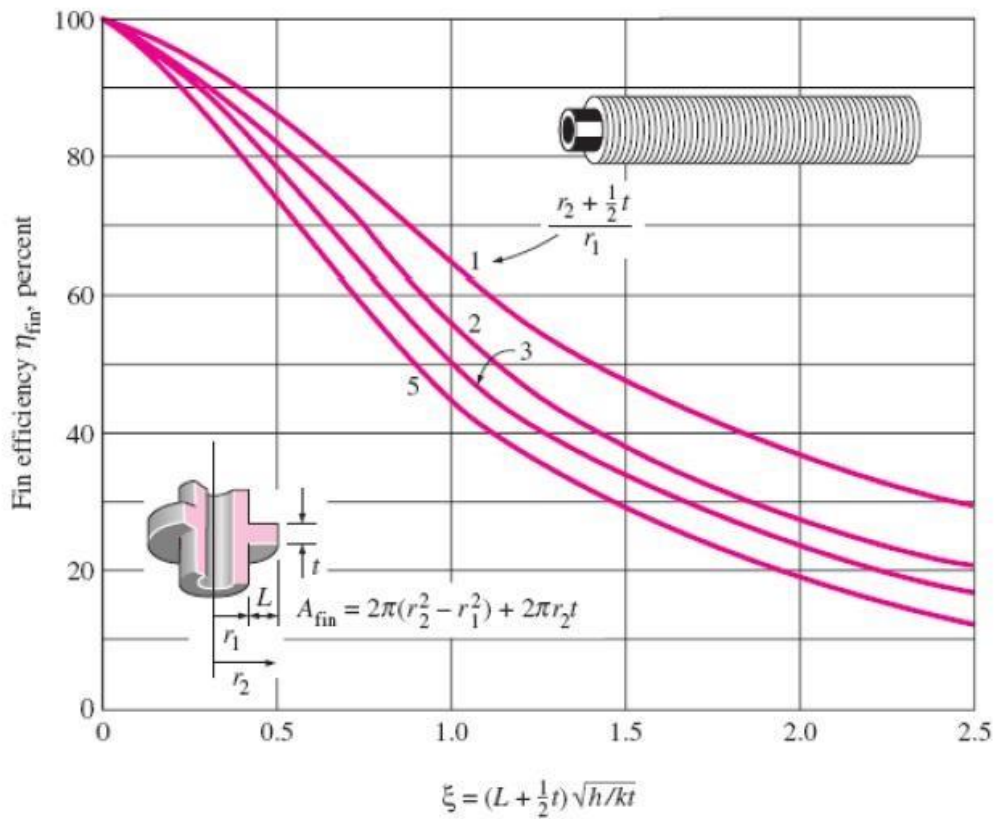
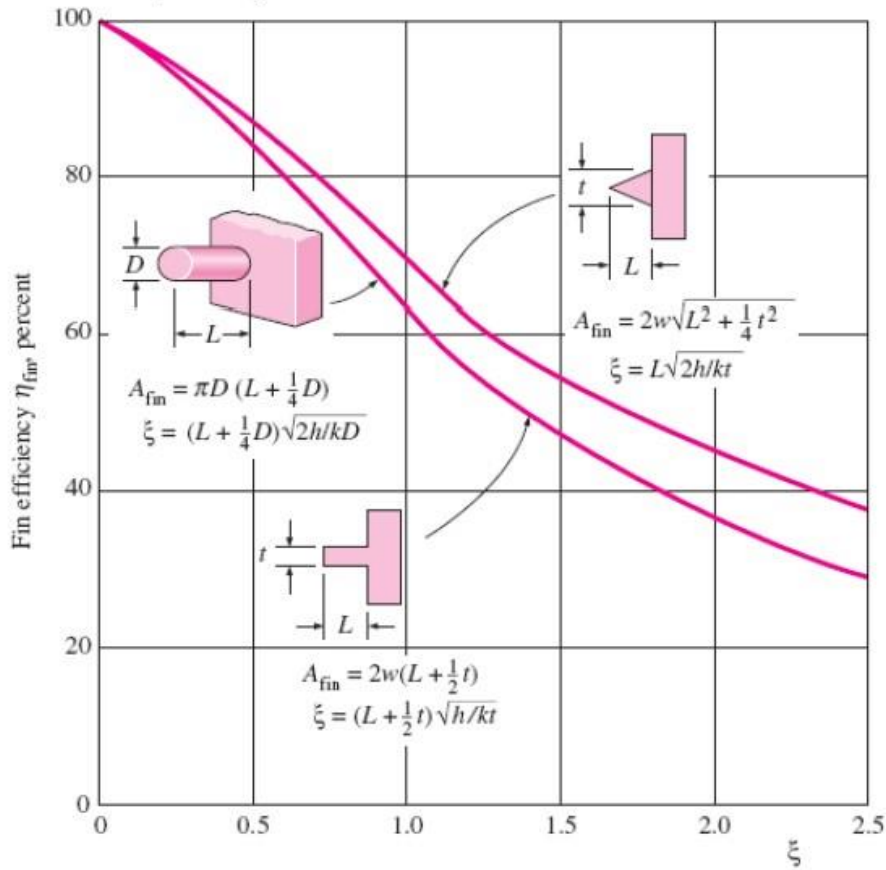
$$\begin{aligned}
 A_{\text{no fin}} &= w \times H \\
 A_{\text{unfin}} &= w \times H - 3 \times (t \times w) \\
 A_{\text{fin}} &= 2 \times L \times w + t \times w \text{ (one fin)} \\
 &= 2 \times L \times w
 \end{aligned}$$

$$\mathcal{E}_{\text{fin, overall}} = \frac{\dot{Q}_{\text{total, fin}}}{\dot{Q}_{\text{total, no fin}}} = \frac{h(A_{\text{unfin}} + \eta_{\text{fin}} A_{\text{fin}})(T_b - T_\infty)}{hA_{\text{no fin}}(T_b - T_\infty)}$$

When determining the rate of heat transfer from a finned surface, we must consider the unfinned portion of the surface as well as the fins. Therefore, the rate of heat transfer for a surface containing n fins can be expressed as

$$\begin{aligned}
 \dot{Q}_{\text{total, fin}} &= \dot{Q}_{\text{unfin}} + \dot{Q}_{\text{fin}} \\
 &= hA_{\text{unfin}}(T_b - T_\infty) + \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty) \\
 &= h(A_{\text{unfin}} + \eta_{\text{fin}} A_{\text{fin}})(T_b - T_\infty)
 \end{aligned}$$

We can also define an **overall effectiveness for a finned surface** as the ratio of the total heat transfer from the finned surface to the heat transfer from the same surface if there were no fins



Proper length of a fin

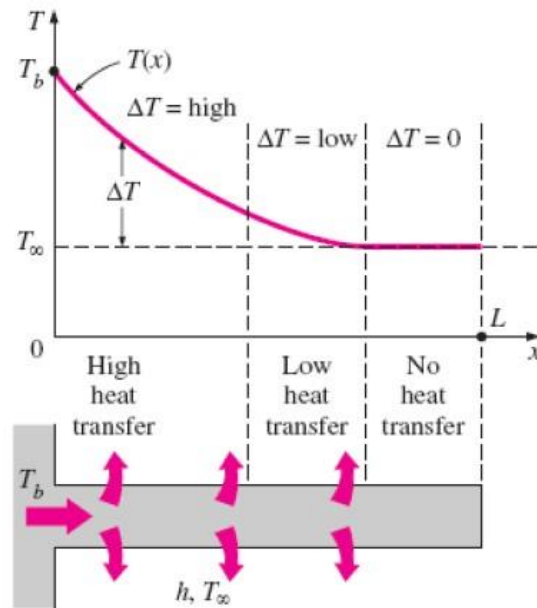
Heat transfer
ratio:

$$\frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{long fin}}} = \frac{\sqrt{hp}kA_c (T_b - T_\infty) \tanh aL}{\sqrt{hp}kA_c (T_b - T_\infty)} = \tanh aL$$

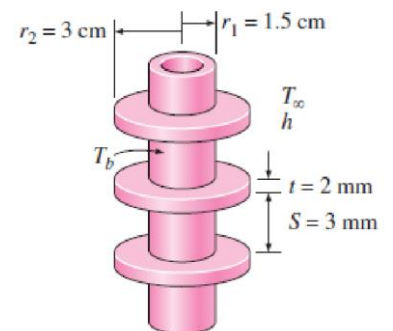
"a" means "m"

The variation of heat transfer from a fin relative to that from an infinitely long fin

aL	$\frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{long fin}}} = \tanh aL$
0.1	0.100
0.2	0.197
0.5	0.462
1.0	0.762
1.5	0.905
2.0	0.964
2.5	0.987
3.0	0.995
4.0	0.999
5.0	1.000



Ex1: Steam in a heating system flows through tubes whose outer diameter is $D_1 = 3$ cm and whose walls are maintained at a temperature of 120°C . Circular aluminum fins ($k = 180 \text{ W/m} \cdot ^\circ\text{C}$) of outer diameter $D_2 = 6$ cm and constant thickness $t = 2$ mm are attached to the tube, as shown in Fig. space between the fins is 3 mm, and thus there are 200 fins per meter length of the tube. Heat is transferred to the surrounding air at $T_\infty = 25^\circ\text{C}$, with a combined heat transfer coefficient of $h = 60 \text{ W/m}^2 \cdot ^\circ\text{C}$. Determine the increase in heat transfer from the tube per meter of its length as a result of adding fins.



Ex2: An aluminum rod ($k = 180 \text{ W/m} \cdot ^\circ\text{C}$) 2 cm in diameter and 12 cm long protrudes from a wall that is maintained at 250°C . The rod is exposed to an environment at 15°C . The convection heat transfer coefficient is $12 \text{ W/m}^2 \cdot ^\circ\text{C}$. Calculate the heat lost by the rod.

Ex3:

A straight rectangular fin ($k = 55 \text{ W/m.K}$) has a length of 3.5 cm, a width of 20 cm and a thickness of 1.4 mm. The fin is exposed to a convection environment at 20°C and $h = 500 \text{ W/m}^2.\text{K}$.

$[q_f = M \tanh mL_c, \text{ where } M = \theta_b \sqrt{hPkA_c}, m^2 = hP/(kA_c) \text{ and } L_c \text{ is the corrected length}]$

- a) For a base temperature of 150°C , calculate the following:
- The maximum possible heat loss
 - The actual heat loss
 - The fin efficiency

