Boolean Function Forms

We will get four Boolean product terms by combining two variables (literals) x and y with logical AND operation. These Boolean product terms are called as **Minterms** or **product terms**. The minterms are $\bar{x}\bar{y},\ \bar{x}y,\ x\bar{y},\ xy$.

Similarly, we will get four Boolean sum terms by combining two variables x and y with logical OR operation. These Boolean sumterms are called as **Maxterms** or **sum terms**. The Maxterms are $x + y,\ x + \bar{y},\ \bar{x} + y,\ \bar{x} + \bar{y}$.

The following table shows the representation of Minterms and Maxterms for 2 variables.

x	y	Minterms	Maxterms
0	0	$m_0 = \bar{x}\,\bar{y}$	$M_0 = x + y$
0	1	$m_1 = \bar{x}\,y$	$M_1 = x + \bar{y}$
1	0	$m_2 = x\,\bar{y}$	$M_2 = \bar{x} + y$
1	1	$m_3 = x\,y$	$M_3 = \bar{x} + \bar{y}$

If the binary variable is '0', then it is represented as complement of variable in Minterm and as the variable itself in Maxterm. Similarly, if the binary variable is '1', then it is represented as complement of variable in Maxterm and as the variable itself in Minterm.

a) Canonical forms

A truth table consists of a set of inputs and outputs. If there are 'n' input variables, then there will be 2^n possible combinations with zeros and ones. So the value of each output variable depends on the combination of input variables. So, each output variable will have '1' for some combination of input variables and '0' for some other combination of input variables. Therefore, we can express each output variable in following two ways.

1- Canonical sum of product (SoP) form
2- Canonical product of sum (PoS) form

1- Canonical SoP form

Canonical SoP form means Canonical Sum of Products form. In this form, each product term **contains all literals**. So, these product terms are nothing but the minterms. Hence, canonical SoP form is also called as **sum of Minterms** form.

- a- identify the minterms for which, the output variable is one
- b- do the logical OR of those minterms in order to get the Boolean expression function corresponding to that output variable.

Example: Drive the canonical SoP form from the following truth table

Inputs			Output	
A	B	C	F	
0	0	0	1	$\overline{A}\overline{B}\overline{C}$
0	0	1	0	
0	1	0	0	
0	1	1	1	$\overline{A}BC$
1	0	0	1	$A\overline{B}\overline{C}$
1	0	1	0	
1	1	0	1	$AB\overline{C}$
1	1	1	1	ABC

Here, the output F is '1' for five combinations of inputs. The corresponding Minterms are $\overline{A}\overline{B}\overline{C}, \overline{A}BC, A\overline{B}\overline{C}, AB\overline{C}, ABC$. By doing logical OR of these five minterms, we will get the Boolean function of output F.

Therefore, the Boolean function of output is,

$$F = \overline{A}\overline{B}\overline{C} + \overline{A}BC + A\overline{B}\overline{C} + AB\overline{C} + ABC$$

This is the **canonical SoP form** of output,F. We can also represent this function in following two notations.

$$F(A,B,C) = m_0 + m_3 + m_4 + m_6 + m_7$$

$$F(A, B, C) = \sum(0, 3, 4, 6, 7)$$

2- Canonical PoS form

In this form, **each sum term contains all literals**. So, these sum terms are nothing but the Maxterms. Hence, canonical PoS form is also called as **product of Maxterms** form.

a- identify the Maxterms for which, the output variable is zero
b- do the logical AND of those Maxterms in order to get the Boolean expression function corresponding to that output variable.

Example: Drive the canonical PoS form from the following truth table

Inputs			Output	
A	**B**	**C**	**F**	
0	**0**	**0**	**1**	
0	**0**	**1**	**0**	$A + B + \overline{C}$
0	**1**	**0**	**0**	$A + \overline{B} + C$
0	**1**	**1**	**1**	
1	**0**	**0**	**1**	
1	**0**	**1**	**0**	$\overline{A} + B + \overline{C}$
1	**1**	**0**	**1**	
1	**1**	**1**	**1**	

Here, the output F is '0' for three combinations of inputs. The corresponding maxterms are $A + B + \overline{C}, \ A + \overline{B} + C, \ \overline{A} + B + \overline{C},$. By doing logical AND of these three maxterms, we will get the Boolean function of output F.

Therefore, the Boolean function of output is,

$$F = (A + B + \overline{C})(A + \overline{B} + C)(\overline{A} + B + \overline{C})$$

This is the **canonical PoS form** of output, F. We can also represent this function in following two notations.

$$F(A, B, C) = M_1. \ M_2. \ M_5$$

$$F(A, B, C) = \prod(1, 2, 5)$$

Note: the sequence of literals(letters) must be the same in truth table and in Boolean equation.

b) Standard forms

We discussed two canonical forms of representing the Boolean outputs. Similarly, there are two standard forms of representing the Boolean outputs. These are the simplified version of canonical forms.

- Standard SoP form: such as $F = AB + \overline{A}C + \overline{B}\overline{C}$
- Standard PoS form: such as $F = (A + C)(A + B + C)(\overline{A} + B)$

The main **advantage** of standard forms is that the number of inputs applied to logic gates can be minimized. Sometimes, there will be reduction in the total number of logic gates required.

Conversion from Canonical SoP form to Standard SoP form

In this form, each product term need not contain all literals. So, the product terms **may** or **may not** be the Minterms. Therefore, the Standard SoP form is the simplified form of canonical SoP form.

We will get Standard SoP form of output variable in two steps.

- Get the canonical SoP form of output variable
- Simplify the above Boolean function, which is in canonical SoP form.

Sometimes, it may not possible to simplify the canonical SoP form. In that case, both canonical and standard SoP forms are same.

Example: convert the canonical SoP expression to standard SoP form

$$F = \overline{A}\overline{B}\overline{C} + AB\overline{C} + ABC$$

Solution:
$F = \overline{A}\overline{B}\overline{C} + AB\overline{C} + ABC$
$F = \overline{A}\overline{B}\overline{C} + AB(\overline{C} + C)$
$F = \overline{A}\overline{B}\overline{C} + AB$ because $\overline{C} + C = 1$

Hence the standard SoP expression of the given function is

$$F = \overline{A}\overline{B}\overline{C} + AB$$

Conversion from standard SoP form to canonical SoP form

The canonical SoP form of a logic function can be obtained by using the following procedure:

1) Check each term in the given logic function. Retain if it is a minterm, continue to examine the next term in the same manner.

2) Examine for the variables that are missing in each product which is not a min term. If the missing variable in the minterm is X, multiply that minterm with (X+X′).

3) Multiply all the products and discard the redundant terms.

Example. Obtain the canonical SoP form of the following function:

$F\ (A,\ B) = A + B$

Solution. The given function contains two variables A and B. The variable B is missing from the first term of the expression and the variable A is missing from the second term of the expression. Therefore, the first term is to be multiplied by $(B + \overline{B})$ and the second term is to be multiplied by $(A + \overline{A})$ as demonstrated below.

$$F(A, B) = A + B$$
$$= A.\mathbf{1} + B.\mathbf{1}$$
$$= A(B + \overline{B}) + B(A + \overline{A})$$
$$= AB + A\overline{B} + BA + B\overline{A}$$
$$= AB + A\overline{B} + B\overline{A} \qquad where \qquad AB + BA = AB$$

Hence the canonical SoP expression of the given function is
$$F(A, B) = AB + A\overline{B} + B\overline{A}$$

Example. Obtain the canonical sum of product form of the following function.
$F\ (A,\ B,\ C) = A + BC$

Solution. Here neither the first term nor the second term is Minterm. The given function contains three variables A, B, and C. The variables B and C are missing from the first term of the expression and the variable A is missing from the second term of the expression. Therefore, the first term is to be multiplied by $(B + \overline{B})$ and $(C + \overline{C})$. The second term is to be multiplied by $(A + \overline{A})$. This is demonstrated below.

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$$F(A, B, C) = A + BC$$
$$= A.1.1 + BC.1$$
$$= A(B + \overline{B})(C + \overline{C}) + BC(A + \overline{A})$$
$$= (AB + A\overline{B})(C + \overline{C}) + BCA + BC\overline{A}$$
$$= ABC + AB\overline{C} + A\overline{B}C + A\overline{B}\overline{C} + BCA + BC\overline{A}$$
$$= ABC + AB\overline{C} + A\overline{B}C + A\overline{B}\overline{C} + BC\overline{A} \quad where \quad ABC + BCA = ABC$$

Hence the canonical SoP expression of the given function is
$$F(A, B, C) = ABC + AB\overline{C} + A\overline{B}C + A\overline{B}\overline{C} + BC\overline{A}$$

Conversion from standard PoS form to canonical PoS form

The canonical product of sums form of a logic function can be obtained by using the following procedure.

1) Check each term in the given logic function. Retain it if it is a maxterm, continue to examine the next term in the same manner.

2) Examine for the variables that are missing in each sum term that is not a maxterm. If the missing variable in the maxterm is X, add that maxterm with $(X\overline{X})$.

3) Expand the expression using the properties and postulates as described earlier and discard the redundant terms.

Example. Obtain the canonical product of the sum form of the following function.
$$F(A, B, C) = (A + \overline{B})(A + C)$$

Solution. Now, in the above expression, C is missing from the first term and B is missing from the second term. Hence $C\overline{C}$ is to be added with the first term and $B\overline{B}$ is to be added with the second term as shown below.

$$F(A, B, C) = (A + \overline{B} + 0)(A + C + 0)$$
$$= (A + \overline{B} + C\overline{C})(A + C + B\overline{B})$$
$$= (A + \overline{B} + C)(A + \overline{B} + \overline{C})(A + C + B)(A + C + \overline{B})$$
$$= (A + \overline{B} + C)(A + \overline{B} + \overline{C})(A + C + B)$$

Hence the canonical PoS expression for the given function is
$$F(A, B, C) = (A + \overline{B} + C)(A + \overline{B} + \overline{C})(A + C + B)$$