1. Find w_{xy} for $w = x^2\cos(2x-3y)$.

$w_y = 3x^2\sin(2x-3y)$ *or* $w_x = -2x^2\sin(2x-3y) + 2x\cos(2x-3y)$

$w_{xy} = -6x^2\cos(2x-3y) + 6x\sin(2x-3y)$

---

2. Find $\nabla\cdot\vec{\boldsymbol{F}}$ for $\vec{\boldsymbol{F}}(x,y,z) = ze^{xz}\hat{\boldsymbol{\imath}} + xe^{xy}\hat{\boldsymbol{\jmath}} + y\,e^{yz}\hat{\boldsymbol{k}}$ at $(1,-1,1)$

$$\nabla = \frac{\partial}{\partial x}\hat{\boldsymbol{\imath}} + \frac{\partial}{\partial y}\hat{\boldsymbol{\jmath}} + \frac{\partial}{\partial z}\hat{\boldsymbol{k}}$$

$$\nabla\cdot\vec{\boldsymbol{F}} = z^2e^{xz} + x^2e^{xy} + y^2e^{yz}$$

$$\nabla\cdot\vec{\boldsymbol{F}}\Big|_{at\ (1,-1,1)} = e + e^{-1} + e^{-1} = e + 2e^{-1}$$

---

3. Evaluate $\int_1^3\int_1^2 (2x+y)\,dx\,dy = \int_1^3 (x^2+xy)\Big|_1^2\,dy$

$$= \int_1^3 (4+2y-1-y)\,dy$$

$$= \int_1^3 (3+y)\,dy = 3y + \frac{y^2}{2}\Big|_1^3$$

$$= 9 + \frac{9}{2} - 3 - \frac{1}{2} = 10$$

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4. Evaluate $\int_0^{\pi/2} \sin^4 x\,dx$

$2m-1=4 \Rightarrow m=\frac{5}{2}$ *and* $2n-1=0 \Rightarrow n=\frac{1}{2}$

$$\int_0^{\pi/2} \sin^4 x\,dx = \frac{1}{2}B\left(\frac{5}{2},\frac{1}{2}\right)$$

$$= \frac{\Gamma\left(\frac{5}{2}\right)\Gamma\left(\frac{1}{2}\right)}{2\Gamma(3)}$$

$$= \frac{\frac{3}{2}\times\frac{1}{2}\sqrt{\pi}\times\sqrt{\pi}}{2\times 2} = \frac{3\pi}{16}$$

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$$\nabla\cdot\vec{\boldsymbol{F}} = z^2e^{xz} + x^2e^{xy} + y^2e^{yz}$$

$$\nabla\cdot\vec{\boldsymbol{F}}\Big|_{at\ (-1,1,1)} = e^{-1} + e^{-1} + e = 2e^{-1} + e$$

---

3. Evaluate $\int_1^2\int_1^3 (x+2y)\,dy\,dx = \int_1^2 (xy+y^2)\Big|_1^3\, dx$

$$= \int_1^2 (3x+9-x-1)\,dx$$

$$= \int_1^2 (2x+8)\,dx = x^2+8x\Big|_1^2$$

$$= 4+16-1-8 = 11$$

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$$\nabla = \frac{\partial}{\partial x}\hat{\boldsymbol{\imath}} + \frac{\partial}{\partial y}\hat{\boldsymbol{\jmath}} + \frac{\partial}{\partial z}\hat{\boldsymbol{k}}$$

$$\nabla \cdot \vec{\boldsymbol{F}} = y^2 e^{xy} + z^2 e^{zy} + x^2 e^{xz}$$

$$\nabla \cdot \boldsymbol{F}^{\rightharpoonup}\Big|_{at\ (1,1,-1)} = \boldsymbol{e} + e^{-1} + e^{-1} = 2e^{-1} + e$$

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3. Evaluate $\int_1^3 \int_1^2 (x + 2y)dydx$

$$= \int_1^3 (xy + y^2)\Big|_1^2 dx$$

$$= \int_1^3 (2x + 4 - x - 1)\, dx$$

$$= \int_1^3 (x + 3)\, dx = \frac{x^2}{2} + 3x\Big|_1^3$$

$$= \frac{9}{2} + 9 - \frac{1}{2} - 3 = 10$$

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4. Evaluate $\int_0^{\pi/2} \cos^4 x\, dx$

$2m - 1 = 4 \;\Rightarrow\; m = \frac{5}{2}$ *and* $2n - 1 = 0 \;\Rightarrow\; n = \frac{1}{2}$

$$\int_0^{\pi/2} \cos^4 x\, dx = \frac{1}{2} B\left(\frac{5}{2}, \frac{1}{2}\right)$$

$$= \frac{\Gamma\left(\frac{5}{2}\right)\Gamma\left(\frac{1}{2}\right)}{2\Gamma(3)}$$

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$$\nabla = \frac{\partial}{\partial x}\hat{\boldsymbol{\imath}} + \frac{\partial}{\partial y}\hat{\boldsymbol{\jmath}} + \frac{\partial}{\partial z}\widehat{\boldsymbol{k}}$$

$$\nabla \cdot \vec{\boldsymbol{F}} = y^2 e^{xy} + z^2 e^{zy} + x^2 e^{xz}$$

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