# Buoyancy and Submergence

## Introduction:

Anybody, floating or immersed completely or partially in a liquid, is acted upon by a ***buoyant force*** equal to the weight of the fluid displaced.

The point through which this force acts is called the ***center of buoyancy*** which is located at the center of gravity of the displaced fluid.

Archimedes principle of fluid displacement has been used to find the volume of an irregular solid.

# Buoyancy and Submergence

**Archimedes principle (287 – 212 B.C.):**

**(the two laws of buoyancy)**

**1. A body submerged in a fluid experiences a vertical buoyant force equal to the weight of the fluid it displaces.**

$$\boxed{F_B = W_{Fluid}}$$

**2. A floating body displaces its own weight in the fluid in which it floats.**

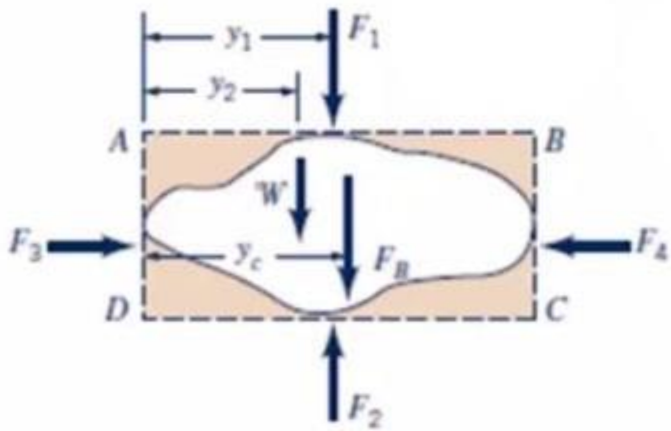
$$\boxed{F_B = \gamma_{Fluid} \, \forall_{body}}$$

## Derivation of the buoyant force ($F_B$):

To show all forces acting on such a body.

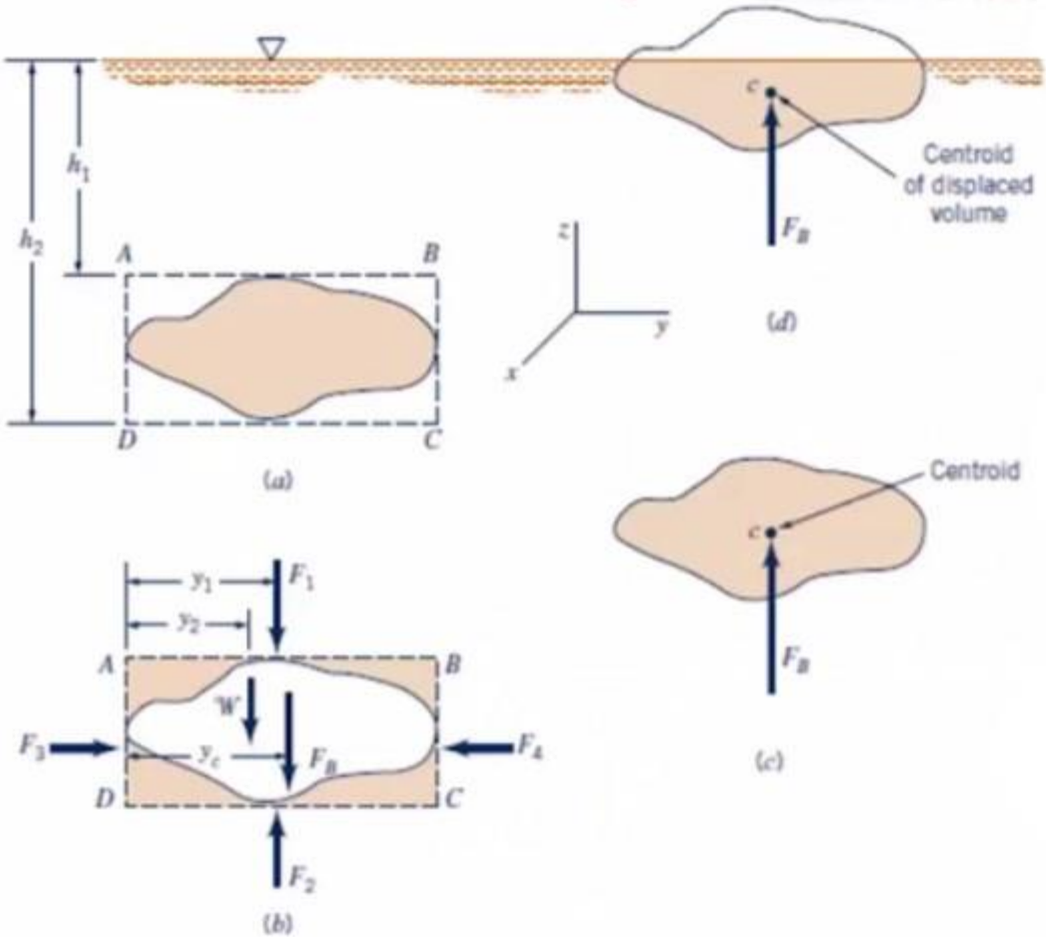
Enclose the body in a *parallelepiped* , draw a free body diagram (FBD) around the *parallelepiped*.

Removed the body as shown in the figure below.



# Buoyancy and Submergence

## Derivation of the buoyant force ($F_B$):




Buoyant force on submerged and floating bodies.

## Derivation of the buoyant force ($F_B$) (cont.):

The forces $F_1$, $F_2$, $F_3$, and $F_4$ are the forces exerted on the plane surfaces of the *parallelepiped*.

For simplicity, the forces in the x-direction are not shown.

W is the weight of the fluid volume represented by the brown shaded area (Parallelepiped minus body).

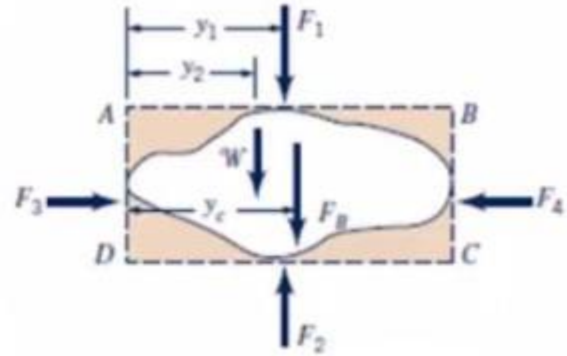
$F_B$ is the force the body is exerting on the fluid.

# Buoyancy and Submergence

## Derivation of the buoyant force ($F_B$) (cont.):

**Thus, at equilibrium:**

$$\boxed{F_B = F_2 - F_1 - W} \quad (9-1)$$




**at constant specific weight of the fluid $\gamma_{Fluid}$**

$$\boxed{F_2 - F_1 = \gamma_{Fluid}(h_2 - h_1)A} \quad (9-2)$$

**A = The horizontal area of the upper**

**(or lower) surface of the parallelepiped**

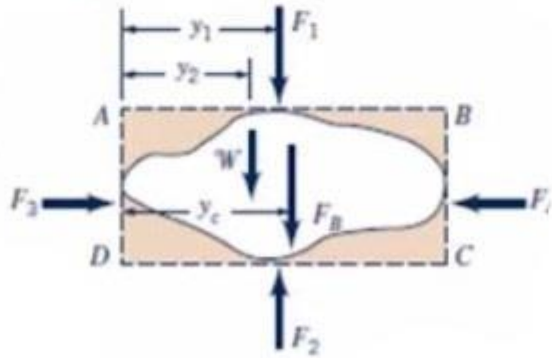
## Derivation of the buoyant force ($F_B$) (cont.):

**Substitute Eq.(9-2) into Eq.(9-1)**

**Thus, Eq(1) can be written as:**

$$F_B = \gamma_{Fluid} \cancel{(h_2 - h_1)A} - \gamma_{Fluid}\left[\cancel{(h_2 - h_1)A} - \forall\right]$$

$$\boxed{F_B = \gamma_{Fluid} \forall_{body}} \quad (9-3)$$




**$\gamma_{Fluid} = \rho_{Fluid}\, g$ = Specific weight of the fluid**

**g = Earth's Standard gravity (9.81 m/sec$^2$)**

**$\forall_{body}$ = the volume of the body (m$^3$ or ft$^3$)**

# Buoyancy and Submergence

**Example:**

Offshore life jacket is shown in the figure.

The jacket must provide a minimum net upward force on the user ($F_u$ = 22 lb).

Consider such a life jacket that uses a foam material with a $\gamma_{Foam}$= 2.0 lb/ft$^3$ for the main floatation material.

The remaining material (cloth, straps, fasteners, etc.) weighs ($W_s$ = 1.3 lb).

Determine the minimum volume of foam needed for this life jacket.

# Buoyancy and Submergence

**Solution:**

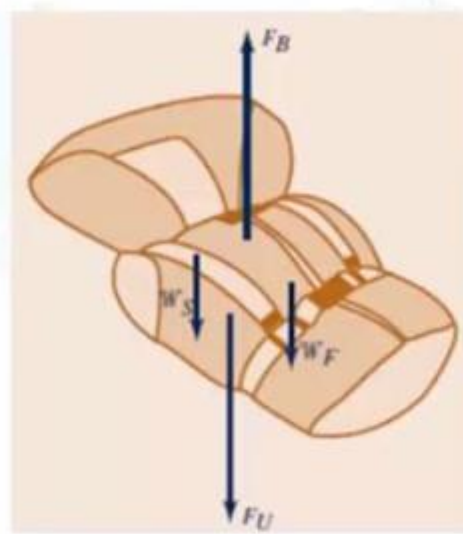
**Given:**

$F_u = 22\text{-lb}$

$W_s = 1.3\ \text{lb}$

$\gamma_{Foam} = 2.0\ \text{lb/ft}^3$

$\gamma_{Seawater} = 64\ \text{lb/ft}^3$



# Buoyancy and Submergence

$F_u = 22\text{-lb}$ , $W_s = 1.3$ lb , $\gamma_{Foam} = 2.0\ lb/ft^3$ , $\gamma_{Seawater} = 64\ lb/ft^3$

$$F_B = W_F + W_s + F_u$$

*Where:*

$$F_B = \gamma_{seawater} \maltese$$

$$W_{foam} = \gamma_{foam} \maltese$$

$$F_B = W_F + W_s + F_u$$

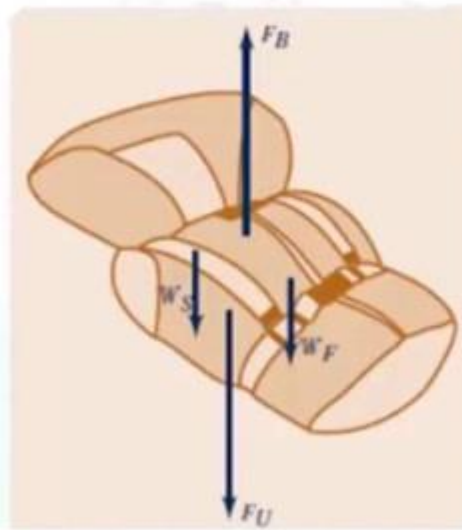
$$(\gamma_{seawater} \maltese) = (\gamma_{foam} \maltese) + (W_s + F_u)$$

$$(\gamma_{seawater} \maltese) - (\gamma_{foam} \maltese) = (W_s + F_u)$$

$$\maltese(\gamma_{seawater} - \gamma_{foam}) = (W_s + F_u)$$

$$\maltese_{foam} = \frac{W_s + F_u}{\gamma_{seawater} - \gamma_{foam}} = \frac{1.3\ lb + 22\ lb}{64\ lb/ft^3 - 2\ lb/ft^3}$$

$$\maltese_{foam} = 0.376\ ft^3$$



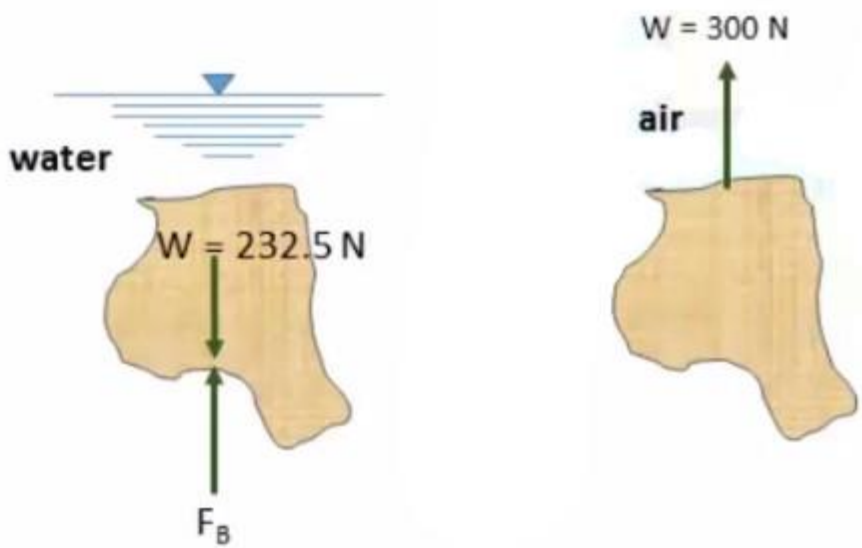
# Buoyancy and Submergence

**Example:**

A piece of irregularly shaped metal weighs 300 N in the air and weighs 232.5 N when completely submerged in water.

Find the volume of the metal.

$\gamma_{water} = 9.7\ kN/m^3$



## Buoyancy and Submergence

**Solution:**

$(F_B) = (W)$

$W_{air} - W_{water} = \gamma_{water} \, \forall$

$300\text{ N} - 232.5\text{ N} = (9.79\text{ kN/m}^3) \,.(1000\text{ N/kN}).(\forall\text{ m}^3)$

$$\forall = \frac{(300 - 232.5)\text{ N}}{(9790)\text{ N/m}^3)} = 0.00689\text{ m}^3$$

**GIVEN** The 0.4-lb lead fish sinker shown in Fig. E2.10*a* is attached to a fishing line as shown in Fig. E2.10*b*. The specific gravity of the sinker is $SG_{\text{sinker}} = 11.3$.

**FIND** Determine the difference between the tension in the line above and below the sinker.

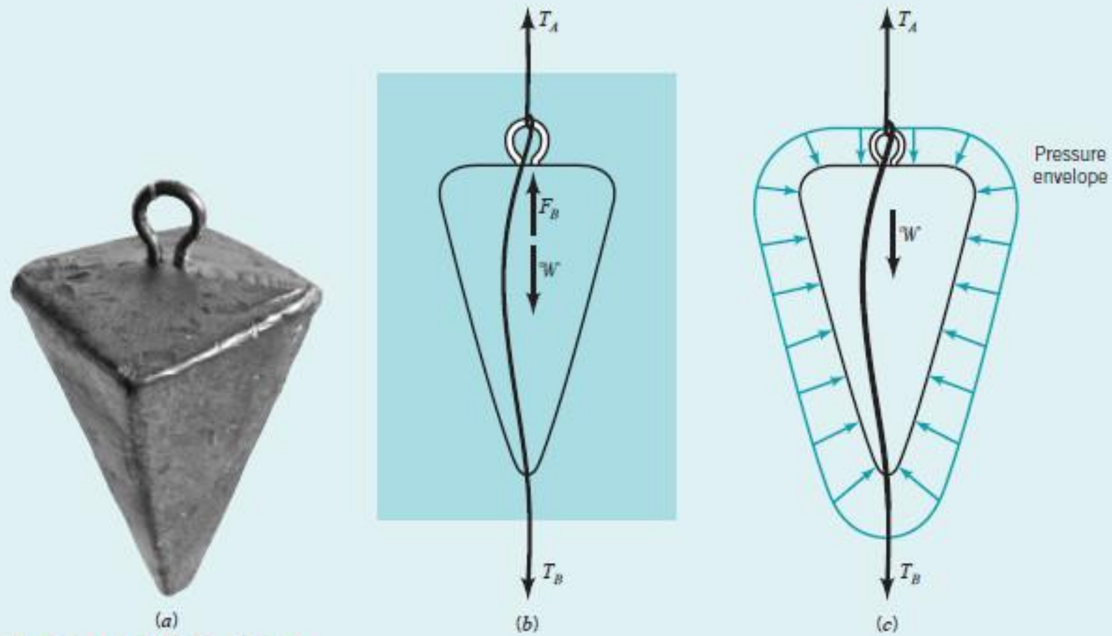



**FIGURE E2.10**

## SOLUTION

A free body diagram of the sinker is shown in Fig. E.10*b*, where $\mathcal{W}$ is the weight of the sinker, $F_B$ is the buoyant force acting on the sinker, and $T_A$ and $T_B$ are the tensions in the line above and below the sinker, respectively. For equilibrium it follows that

$$T_A - T_B = \mathcal{W} - F_B \tag{1}$$

Also,

$$\mathcal{W} = \gamma_{\text{sinker}} \mathcal{V} = \gamma SG_{\text{sinker}} \mathcal{V} \tag{2}$$

where $\gamma$ is the specific weight of water and $\mathcal{V}$ is the volume of the sinker. From Eq. 2.22,

$$F_B = \gamma \mathcal{V} \tag{3}$$

By combining Eqs. 2 and 3 we obtain

$$F_B = \mathcal{W}/SG_{\text{sinker}} \tag{4}$$

Hence, from Eqs. 1 and 4 the difference in the tensions is

$$T_A - T_B = \mathcal{W} - \mathcal{W}/SG_{\text{sinker}} = \mathcal{W}[1 - (1/SG_{\text{sinker}})] \tag{5}$$

$$= 0.4 \text{ lb}\,[1 - (1/11.3)] = 0.365 \text{ lb} \qquad \textbf{(Ans)}$$

**COMMENTS** Note that if the sinker were raised out of the water, the difference in tension would equal the entire weight of the sinker ($T_A - T_B = 0.4$ lb) rather than the 0.365 lb when it is in the water. Thus, since the sinker material is significantly heavier than water, the buoyant force is relatively unimportant. As seen from Eq. 5, as $SG_{\text{sinker}}$ becomes very large, the buoyant force becomes insignificant, and the tension difference becomes nearly equal to the weight of the sinker. On the other hand, if $SG_{\text{sinker}} = 1$, then $T_A - T_B = 0$ and the sinker is no longer a "sinker." It is neutrally buoyant and no external force from the line is required to hold it in place.

# Buoyancy and Submergence (2)

## Introduction:

The stability of a submerged or floating bodies is strongly related to the position of the center of gravity with respect to the position of the center of buoyancy.

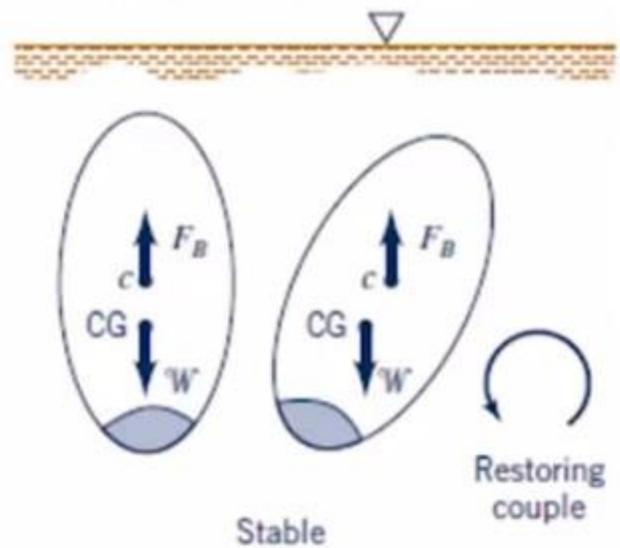
For stability of a *submerged body or floating cylinders or spheres* the center of gravity of the body must lie directly below the center of buoyancy of the displaced liquid.

Stability

# Buoyancy and Submergence (2)

**Stability of a completely submerged body:**

**The location of the center of gravity (CG) in regard to the center of buoyancy (c) is very important.**

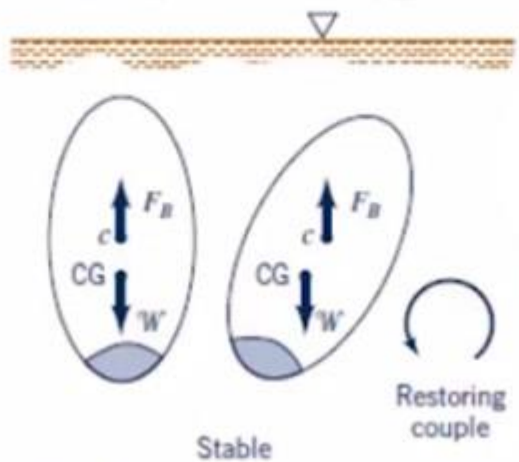



**(10-a): Stability of a completely submerged Body – center of**

# Buoyancy and Submergence (2)

1. Location of the center of buoyancy (c) in regard to the center of gravity (CG):

Such arrangement will create a restoring couple formed by the weight (W) and the buoyant force, ($F_B$) which causes the body to rotate back to its original position.

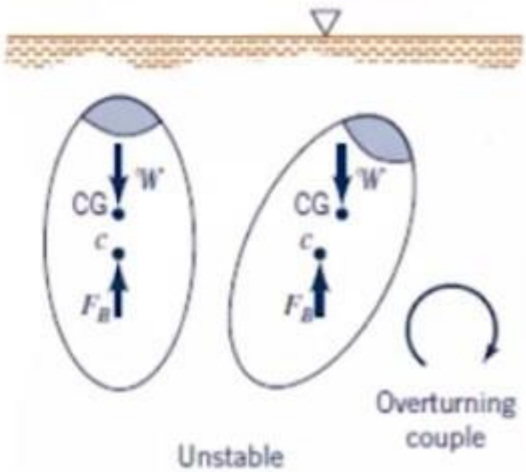



(10-a): Stability of a completely submerged Body – center of gravity below center of buoyancy

# Buoyancy and Submergence (2)

2. If the center of buoyancy (c) is below the center of gravity (CG):

The center of gravity is now above the center of buoyancy as shown.

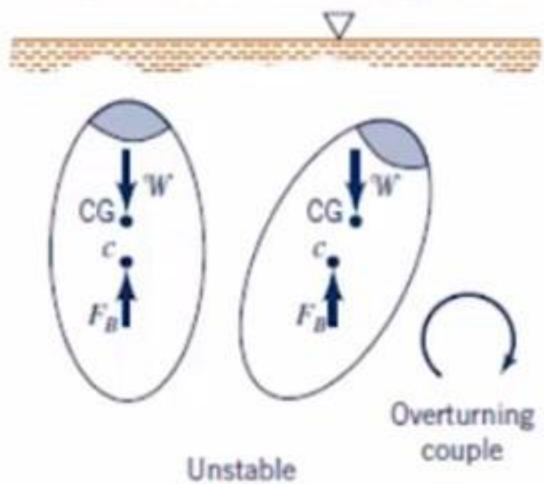



(10-b) Stability of a completely submerged Body – center of gravity above center of buoyancy

# Buoyancy and Submergence (2)

A third condition, if the center of buoyancy and center of gravity are coincident.

The body neutrally stable, that is, has the tendency for neither restoring nor overturning.



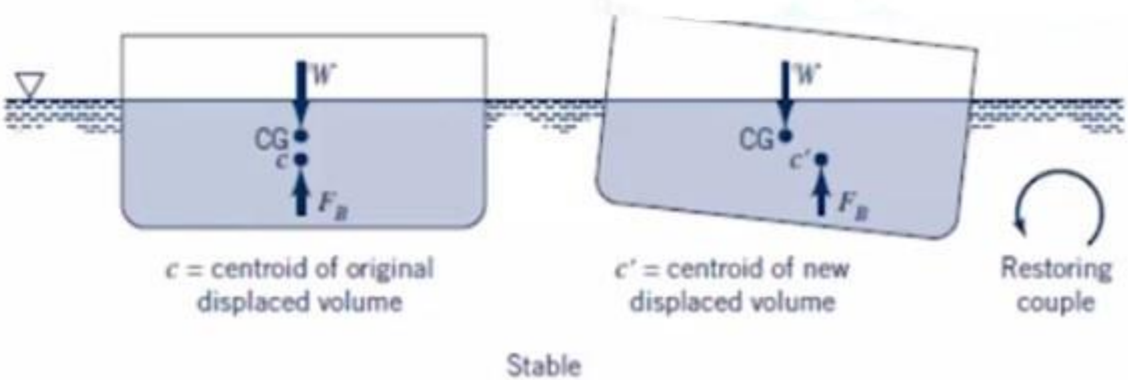

(10-b) Stability of a completely submerged Body – center of gravity above center of buoyancy

# Buoyancy and Submergence (2)

## Stability of floating bodies:

1. Wide flat-bottom bodies:

Floating bodies are more sensitive to instability problems then the completely submerged bodies due to the location of the center of buoyancy (c) with respect to the center of gravity (CG)




Stability of a floating body (stable configuration)

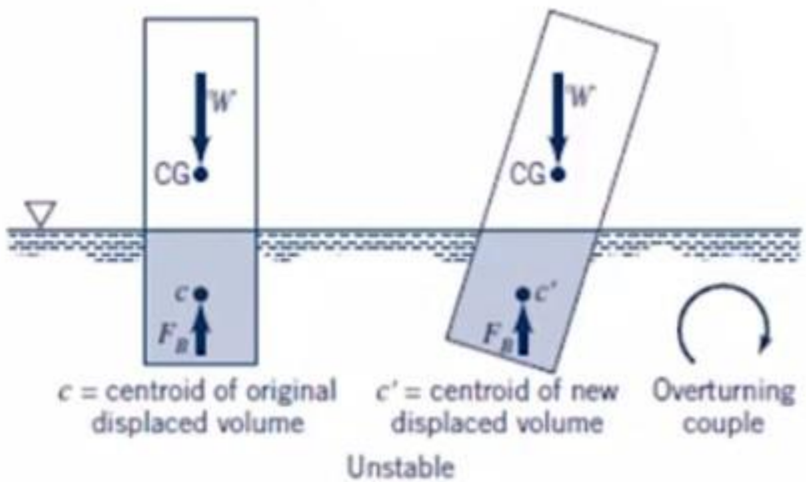
## Buoyancy and Submergence (2)

As the body rotates, the buoyant force ($F_B$), shifts to a new position to pass through the centeroid of the newly formed displaced volume. The buoyant force with the weight (W), will form a restoring couple which will return the body to its original stable position. The stability of the body was partially due to the geometric shape of the flat-bottom boat.

# Buoyancy and Submergence (2)

<u>2. Tall slender bodies</u>:

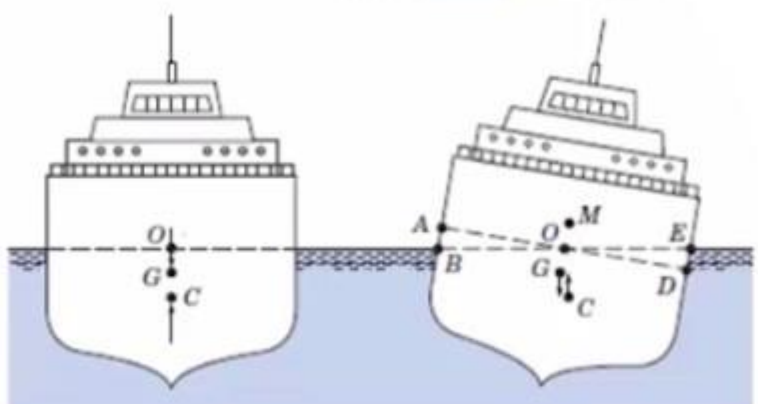
For relatively tall, slender bodies, a small rotational displacement can cause both the buoyant force ($F_B$) and the weight (W) to form an overturning couple as shown.



# Buoyancy and Submergence (2)

The stability considerations are of great importance to engineers in the design of ships and submarines etc.

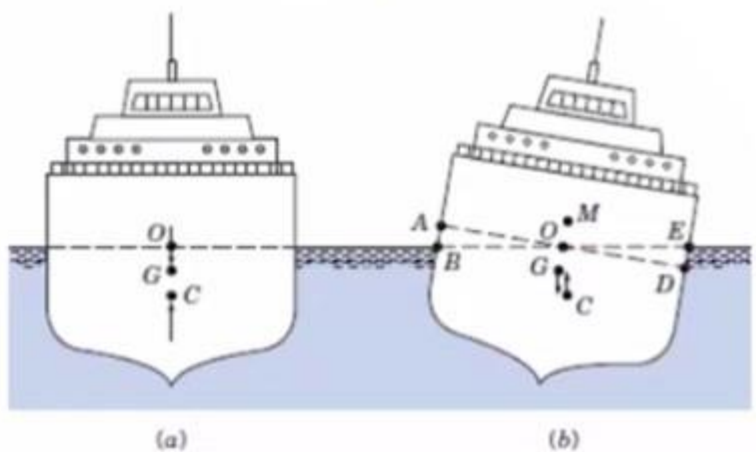
The stability of a ship is its tendency not to overturn when it is in a non-equilibrium position, is indicated by the metacenter (M).




(a) (b)

# Buoyancy and Submergence (2)

The metacenter (M) is a point at which a vertical line through the center of buoyancy (c) intersect the rotated position of the line through the center of gravity (CG) and the center buoyancy (c) for the equilibrium condition (C`G).

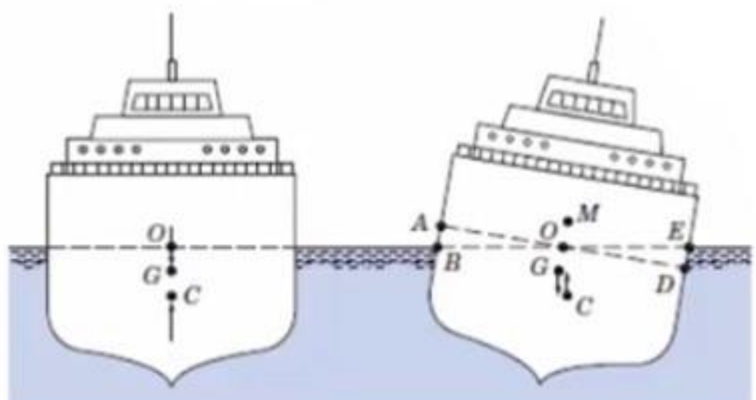


# Buoyancy and Submergence (2)

The ship is stable only if the metacenter (M) is above the center of gravity (CG).

Why?

Because of the resulting moment for this condition will create restoring couple tends to right up the ship.




(a) (b)

# Buoyancy and Submergence (2)

The distance between the ships metacenter (M) and the center of gravity is called the *metacentric height* designated by (GM) which is a measure of the degree of stability or instability of a ship. If (GM) is negative (-), the ship is unstable.

# Buoyancy and Submergence (2)

$$\boxed{GM = \frac{I_{00}}{\forall} - C'G'} \qquad (10-1)$$

GM = metacentric height (m or ft)

= degree of stability (±)

$I_{00}$ = moment of inertia of the area defined by the water line ($m^4$ or $ft^4$), where

$$I_{00} = \frac{1}{12} b a^3$$

$\forall$ = volume of displaced fluid ($m^3$ or $ft^3$)

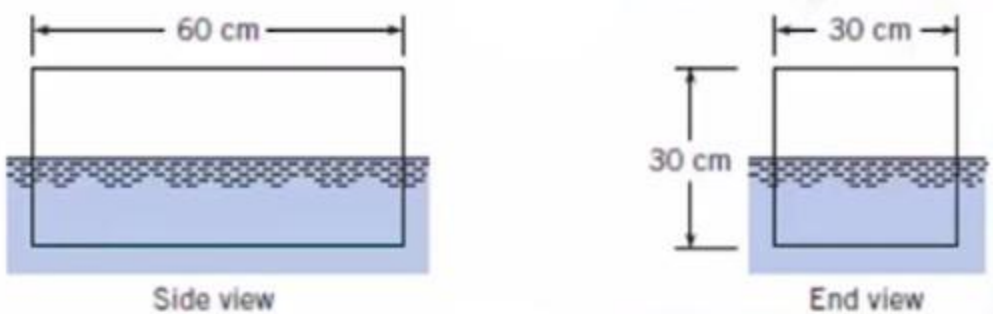
C`G` = distance between center of buoyancy (c) and center of gravity (CG) at equilibrium (m or ft).

# Buoyancy and Submergence (2)

**Example:**

A block of wood 30 cm square in cross section and 60 cm long weighs 318 N.

Will the block float with sides vertical?



# Buoyancy and Submergence (2)

**Solution:**

**1. Determine the depth of submergence of the block (d).**

**This is calculated by applying the equation of equilibrium in the y-direction.**

$$\sum F_y = 0 \Rightarrow -\text{weight} + \text{buoyant force} = 0$$

$$-W + F_B = 0$$

$$\Rightarrow -W + \gamma_{water} \forall = 0$$

$$\Rightarrow -318 + (9810\frac{N}{m^3}) \cdot (0.3\,m \times 0.6\,m \times d) = 0$$

$$\Rightarrow -318 = -1765.8\,d$$

# Buoyancy and Submergence (2)

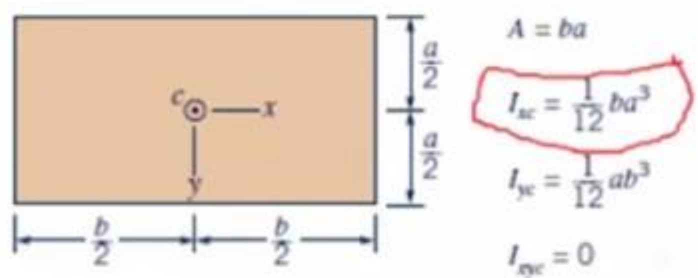
**Solution (cont.):**

$$\Rightarrow d = \frac{-318}{-1765.8} = 0.18\,\text{m} = 18\,\text{cm}$$

$$\Rightarrow \text{Submergence of the block} \quad \boxed{d = 18\,\text{cm}}$$

**2. Determine the stability of the block about the longitudinal axis.**

$$GM = \frac{I_{xx}}{\forall} - C`G`$$




(a) Rectangle

*Where:*

$$I_{xx} = \frac{1}{12} ba^3 \Rightarrow I_{xx} = \frac{1}{12}(60)\cdot(30)^3$$

$$I_{xx} = 135000\,cm^4$$

**Solution (cont.):**

$$\rlap{-}V = d * b * a = 18\ cm * 60\ cm * 30\ cm$$

$$\rlap{-}V = 32400\ cm^4$$

$$C`G` = CG - c \Rightarrow C`G` = \frac{30\ cm}{2} - \frac{18\ cm}{2}$$

$$C`G` = (15\text{-}9)$$

$$C`G` = 6\ cm$$

$$GM = \frac{I_{xx}}{\rlap{-}V} - C`G` \Rightarrow GM = \frac{135000\ cm^4}{32400\ cm^3} - 6\ cm$$

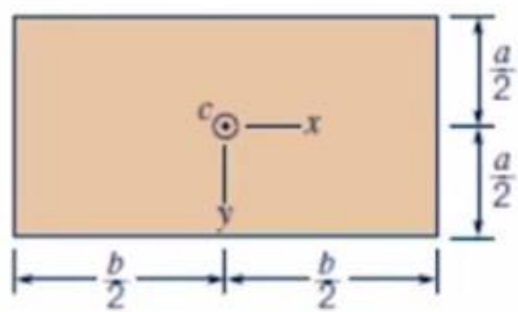
$$GM = -1.833\ cm$$

# Buoyancy and Submergence (2)

**Solution (cont.):**

**The negative sign (-) of the metacentric height indicates that the block is *unstable* about the *longitudinal axis*.**

**Thus, any external force that strikes the block will overturn it.**




$A = ba$

$I_{xc} = \frac{1}{12} ba^3$

$I_{yc} = \frac{1}{12} ab^3$

$I_{xyc} = 0$

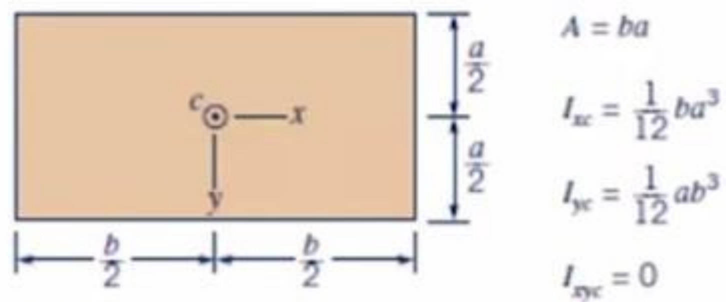
(a) Rectangle

# Buoyancy and Submergence (2)

**Solution (cont.):**

**Now we will check if the block is stable about its *transverse axis*.**

$$GM = \frac{I_{yc}}{\forall} - C`G`$$



(a) Rectangle

*Where:*

$$I_{yc} = \frac{1}{12}ab^3 \Rightarrow I_{yc} = \frac{1}{12}(30)\cdot(60)^3$$

$$I_{yc} = 540000cm^4$$

# Buoyancy and Submergence (2)

$$\forall = d * a * b = 18\ cm * 30\ cm * 60\ cm$$

$$\forall = 32400\ cm^3$$

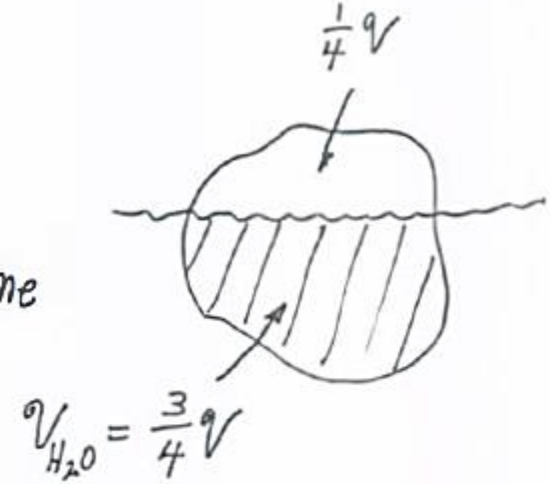
$$C`G` = CG - c \Rightarrow C`G` = \frac{30\ cm}{2} - \frac{18\ cm}{2}$$

$$C`G` = (15\text{-}9) = 6\ cm$$

$$GM = \frac{I_{yc}}{\forall} - C`G` \Rightarrow GM = \frac{540000\ cm^4}{32400\ cm^3} - 6\ cm$$

$$GM = -16.67 - 6 = 10.67\ cm$$

A freshly cut log floats with one fourth of its volume protruding above the water surface. Determine the specific weight of the log.




$\mathcal{V}$ = log volume

$F_B = \mathcal{W}$ or

$$\gamma_{H_2O}\,\mathcal{V}_{H_2O} = \gamma_{log}\,\mathcal{V}$$

Thus,

$$\gamma_{log} = \gamma_{H_2O}\,\frac{\mathcal{V}_{H_2O}}{\mathcal{V}} = \gamma_{H_2O}\,\frac{\frac{3}{4}\mathcal{V}}{\mathcal{V}}$$

or

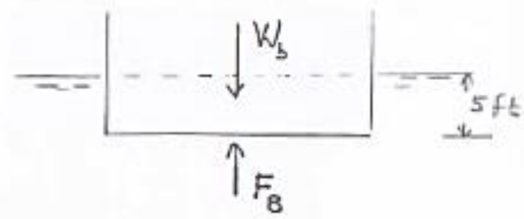
$$\gamma_{log} = \frac{3}{4}\,\gamma_{H_2O} = \frac{3}{4}\left(62.4\,\frac{lb}{ft^3}\right) = \underline{\underline{46.8\,\frac{lb}{ft^3}}}$$

A river barge, whose cross section is approximately rectangular, carries a load of grain. The barge is 28 ft wide and 90 ft long. When unloaded its draft (depth of submergence) is 5 ft, and with the load of grain the draft is 7 ft. Determine: (a) the unloaded weight of the barge, and (b) the weight of the grain

(a) For equilibrium,

$$\sum F_{vertical} = 0$$

so that



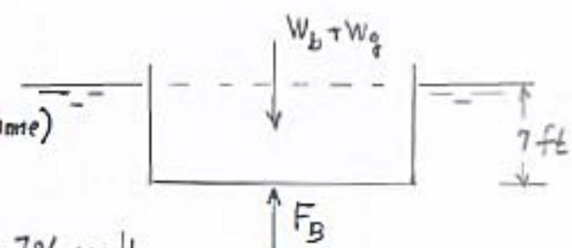

$W_b$ ~ weight of barge (unloaded)

$$W_b = F_B = \gamma_{H_2O} \times (\text{submerged volume})$$

$$= \left(62.4 \frac{lb}{ft^3}\right)(5\ ft \times 28\ ft \times 90\ ft)$$

$$= \underline{\underline{786{,}000\ lb}}$$

(b) $\sum F_{vertical} = 0$




$$W_b + W_g = F_B = \gamma_{H_2O} \times (\text{submerged volume})$$

$$W_g = \left(62.4 \frac{lb}{ft^3}\right)(7\ ft \times 28\ ft \times 90\ ft) - 786{,}000\ lb$$

$W_g$ ~ weight of grain

$$= \underline{\underline{315{,}000\ lb}}$$

**2.99** A tank of cross-sectional area $A$ is filled with a liquid of specific weight $\gamma_1$ as shown in Fig. P2.99*a*. Show that when a cylinder of specific weight $\gamma_2$ and volume $\mathcal{V}$ is floated in the liquid (see Fig. P2.99*b*), the liquid level rises by an amount $\Delta h = (\gamma_2/\gamma_1)\,\mathcal{V}/A$.

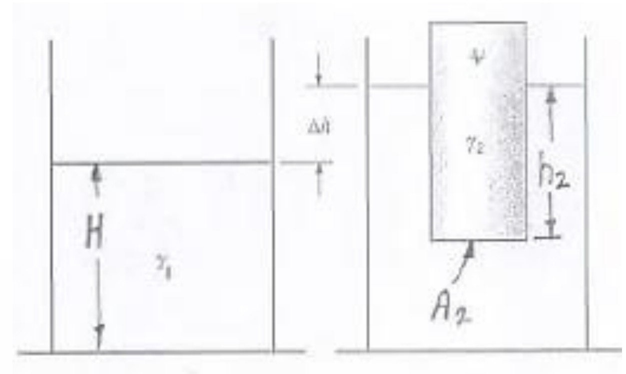



FIGURE P2.99

$W$ = weight of cylinder $= \gamma_2 \mathcal{V}$

For equilibrium,

$W$ = weight of liquid displaced $= \gamma_1 h_2 A_2 = \gamma_1 \mathcal{V}_2$ where $\mathcal{V}_2 = h_2 A_2$

Thus,

$\gamma_2 \mathcal{V} = \gamma_1 \mathcal{V}_2$, or

$$\mathcal{V}_2 = \frac{\gamma_2}{\gamma_1}\mathcal{V}$$

However, the final volume within the tank is equal to the initial volume plus the volume, $\mathcal{V}_2$, of the cylinder that is submerged. That is,

$$(H+\Delta h)A = HA + \mathcal{V}_2$$

or

$$\Delta h = \frac{\mathcal{V}_2}{A} = \underline{\underline{\frac{\gamma_2}{\gamma_1}\frac{\mathcal{V}}{A}}}$$