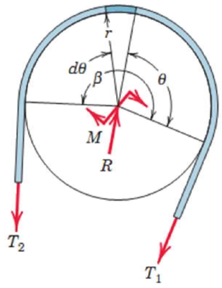
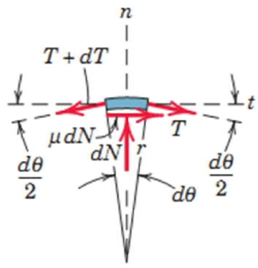
Flexible Belts

The impending slippage of flexible cables, belts, and ropes over sheaves and drums is important in the design of belt drives of all types, band brakes, and hoisting rigs.

Figure a, shows a drum subjected to the two belt tensions T1 and T2, the torque M necessary to prevent rotation, and a bearing reaction R. With M in the direction shown, T2 is greater than T1. The free-body diagram of an element of the belt of length rdθ is shown in figure b. We analyze the forces acting on this differential element analyzed by establishing the equilibrium of the element, in a manner similar to that used for other variable-force problems. The tension increases from T at the angle θ to T+dT at the angle θ+dθ. The normal force is a differential dN, since it acts on a differential element of area. Likewise the friction force, which must act on the belt in a direction to oppose slipping, is a differential and is μdN for impending motion.



(a)



(b)

Equilibrium in the t-direction gives

$$\sum F_t = 0 \Rightarrow T\underbrace{\cos\frac{d\theta}{2}}_{=1} + \mu dN = (T + dT)\underbrace{\cos\frac{d\theta}{2}}_{=1} \Rightarrow T + \mu dN = T + dT \Rightarrow$$

$\mu dN = dT \ldots 1$

Equilibrium in the n-direction requires that

$$\sum F_n = 0 \Rightarrow dN = (T + dT)\underbrace{\sin\frac{d\theta}{2}}_{=\frac{d\theta}{2}} + T\underbrace{\sin\frac{d\theta}{2}}_{=\frac{d\theta}{2}} \Rightarrow dN = T\frac{d\theta}{2} + \underbrace{dT\frac{d\theta}{2}}_{\text{ignored}} + T\frac{d\theta}{2} \Rightarrow$$

$dN = Td\theta \ldots 2$

Combining the two equilibrium relations gives

$$\frac{dT}{T} = \mu d\theta$$

$$\int_{T_1}^{T_2} \frac{dT}{T} = \int_0^{\beta} \mu d\theta \Rightarrow [\ln T]_{T_1}^{T_2} = [\mu\theta]_0^{\beta} \Rightarrow \ln T_2 - \ln T_1 = \mu\beta \Rightarrow \ln\frac{T_2}{T_1} = \mu\beta \Rightarrow e^{\ln\frac{T_2}{T_1}} = e^{\mu\beta} \Rightarrow$$

$$\frac{T_2}{T_1} = e^{\mu\beta} \ldots 3$$

T2: greater force

T1: smaller force

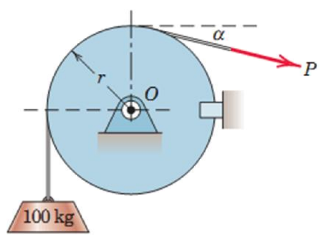
μ: coefficient of static friction.

β: total angle of belt contact and must be expressed in radians. If a rope were wrapped around drum n times, the angle β would be $2\pi n$ radians

Equation 3 holds equally well for a noncircular section where the total angle of contact is β. This conclusion is evident from the fact that the radius r of the circular drum in Fig. does not enter into the equations for the equilibrium of the differential element of the belt.

SAMPLE PROBLEM 6/9

A flexible cable which supports the 100-kg load is passed over a fixed circular drum and subjected to a force P to maintain equilibrium. The coefficient of static friction μ between the cable and the fixed drum is 0.30. (a) For α=0, determine the maximum and minimum values which P may have in order not to raise or lower the load. (b) For P =500 N, determine the minimum value which the angle α may have fore the load begins to slip.

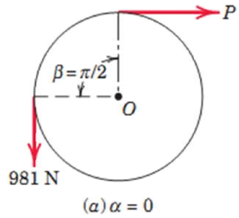


Solution:

(a)

W=100*9.81=981N

With α=0, $\beta = 90° = \frac{\pi}{2}\ rad.$



For impending upward motion of the load, $T_2 = P_{max}$, $T_1 = W = 981N$

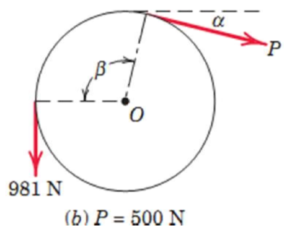
$$\frac{T_2}{T_1} = e^{\mu\beta} \Rightarrow \frac{P_{max}}{981} = e^{0.3*\frac{\pi}{2}} \Rightarrow P_{max} = 1572\ N$$

For impending downward motion of the load, $T_2 = 981N$ and $T_1 = P_{min}$

$$\frac{T_2}{T_1} = e^{\mu\beta} \Rightarrow \frac{981}{P_{min}} = e^{0.3*\frac{\pi}{2}} \Rightarrow P_{min} = 612\ N$$

(b)

$T_2 = 981N\ ,\ T_1 = P = 500\ N$



$$\frac{T_2}{T_1} = e^{\mu\beta} \Rightarrow \frac{981}{500} = e^{0.3\beta} \Rightarrow 1.962 = e^{0.3\beta} \Rightarrow \ln 1.962 = \ln e^{0.3\beta}$$

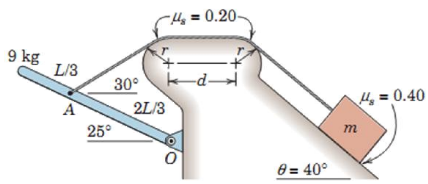
$$\Rightarrow \ln 1.962 = 0.3\beta \Rightarrow \ \beta = \frac{\ln 1.962}{0.3} = 2.25\ rad.$$

$$\beta_{degree} = 2.25 * \frac{180}{\pi} = 128.7°$$

$$\beta = 90° + \alpha \Rightarrow 128.7° = 90° + \alpha \Rightarrow \alpha = 38.7°$$

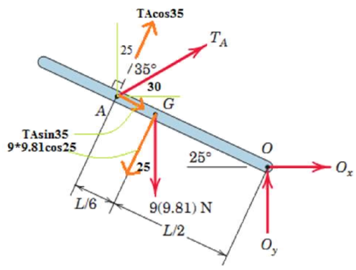
SAMPLE PROBLEM 6/10

Determine the range of mass *m* over which the system is in static equilibrium. The coefficient of static friction between the cord and the upper curved surface is 0.20, while that between the block and the incline is 0.40. Neglect friction at the pivot O.



Solution:

$\circlearrowright \sum M_o = 0 \Rightarrow T_A \cos 35 * \frac{2}{3}L - 9 * 9.81 \cos 25 * \frac{L}{2} = 0 \Rightarrow T_A = 73.3\ N$

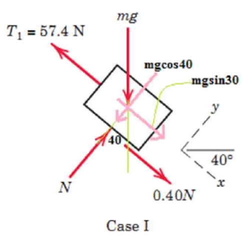


I. Motion of m impends up the incline.

$\therefore T_A = P_{max} = 73.3\ N$

$$\beta = (40 + 30)\frac{\pi}{180} = 1.22\ rad.$$

$$\frac{T_2}{T_1} = e^{\mu\beta} \Rightarrow \frac{73.3}{T_1} = e^{0.2*1.22} \Rightarrow T_1 = 57.4\ N$$



$$\nearrow \sum F_y = 0 \Rightarrow N - mg\cos 40 = 0 \Rightarrow N - mg\cos 40 = 0 \Rightarrow N = 0.76mg$$

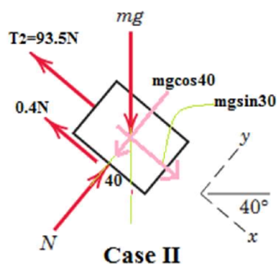
$$\nwarrow \sum F_x = 0 \Rightarrow T_1 - mg\sin 40 - \mu N = 0$$

$$\Rightarrow 57.4 - mg\sin 40 - 0.4 * 0.76mg = 0 \Rightarrow m = 6.16\ kg$$

II. Motion of m impends down the incline.

$\therefore T_A = P_{min} = 73.3\ N$

$$\frac{T_2}{T_1} = e^{\mu\beta} \Rightarrow \frac{T_2}{73.3} = e^{0.2*1.22} \Rightarrow T_2 = 93.5\ N$$



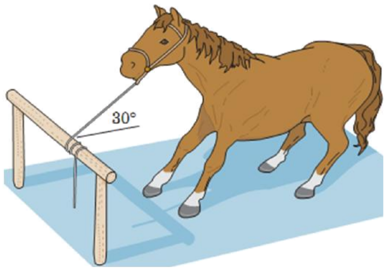
$$\nearrow \sum F_y = 0 \Rightarrow N = 0.76mg$$

$$\nwarrow \sum F_x = 0 \Rightarrow T_2 - mg\sin 40 + \mu N = 0$$

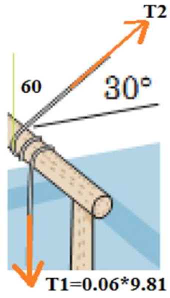
$$\Rightarrow 93.5 - mg\sin 40 + 0.4 * 0.76mg = 0 \Rightarrow m = 28.3kg$$

So the requested range is $6.16 \le m \le 28.3\ \ kg.$

Prob. 6/104



In western movies, cowboys are frequently observed hitching their horses by casually winding a few turns of the reins around a horizontal pole and letting the end hang free as shown—no knots! If the freely hanging length of rein weighs 0.06kg and the number of turns is as shown, what tension T does the horse have to produce in the direction shown in order to gain freedom? The coefficient of friction between the reins and wooden pole is 0.70.



Solution:

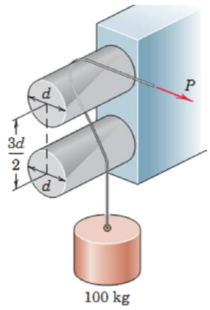
$$T_1 = 0.06 * 9.81 = 0.5886\ N$$

$$\beta = 2 * 2\pi + 60 * \frac{\pi}{180} = 13.61\ rad.$$

$$\frac{T_2}{T_1} = e^{\mu\beta} \Rightarrow \frac{T_2}{0.5886} = e^{0.7*13.61} \Rightarrow T_2 = T = 8079N \cong 8.1\ kN$$

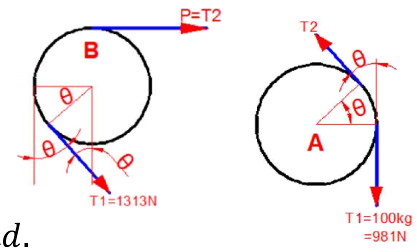
Prob. 6/105

Calculate the horizontal force P required to raise the 100-kg load. The coefficient of friction between the rope and the fixed bars is 0.40.



Solution:

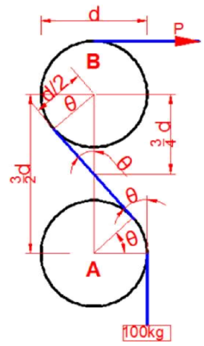
$$\theta = \sin^{-1}\frac{d/2}{\frac{3}{4}d} = 41.8°$$



Pulley A

$$\beta = \theta = 41.8 * \frac{\pi}{180} = 0.7295\ rad.$$

$$\frac{T_2}{T_1} = e^{\mu\beta} \Rightarrow \frac{T_2}{100 * 9.81} = e^{0.4*0.7295} \Rightarrow T_2 = 1313\ N$$



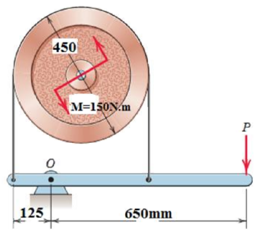
Pulley B

$$\beta = 90 + \theta = (90 + 41.8) * \frac{\pi}{180} = 2.3\ rad.$$

$$\frac{T_2}{T_1} = e^{\mu\beta} \Rightarrow \frac{P}{1313} = e^{0.4*2.3} \Rightarrow P = 3.3\ kN$$

Prob. 6/111

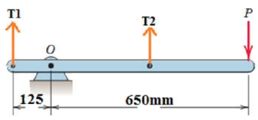
A counterclockwise moment 150 N.m is applied to the flywheel. If the coefficient of friction between the band and the wheel is 0.20, compute the minimum force P necessary to prevent the wheel from rotating.



Solution:

$$\circlearrowleft \sum M_o = 0 \Rightarrow 650P + 125T_1 - (450 - 125)T_2 = 0 \Rightarrow$$

$$650P + 125T_1 - 325T_2 = 0 \ldots\ldots 1$$

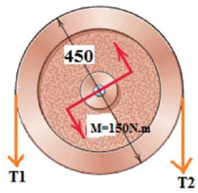


$$\beta = \pi$$

$$\frac{T_2}{T_1} = e^{\mu\beta} \Rightarrow \frac{T_2}{T_1} = e^{0.2\pi} \Rightarrow \frac{T_2}{T_1} = 1.874 \ldots\ldots 2$$

Applied Torque= $150 * 10^3 N.mm$

$$\text{Resisting Torque} = (T_2 - T_1)r \ldots 3 \Rightarrow \left(T_2 - \frac{T_2}{1.874}\right)\frac{450}{2} \ldots\ldots 3 + 2$$



$$M_{applied} = M_{resisting}$$

$$150 * 10^3 = \left(T_2 - \frac{T_2}{1.874}\right)\frac{450}{2} \Rightarrow T_2 = 1429N$$

$$T_1 = \frac{T_2}{1.874} = \frac{1429}{1.874} \Rightarrow T_1 = 762N$$

$$650P + 125T_1 - 325T_2 = 0 \Rightarrow 650P + 125 * 762 - 325 * 1429 = 0 \Rightarrow P = 568N$$