

The Quantum Harmonic Oscillator

The harmonic oscillator in classical physics is a physical system that describes a particle moving under the influence of a restoring force that is directly proportional to its displacement from the equilibrium position and acts in the opposite direction.

This force can be expressed by the relation: $F = -kx$, where k represents the force constant and x is the displacement. The potential energy of this system has a quadratic form given by: $V_x = \frac{1}{2} k x^2$

The harmonic oscillator describes continuous oscillatory motion, where the particle can have any value of energy without restrictions. Its total energy is the sum of kinetic and potential energy: $E = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$

In quantum mechanics, however, the harmonic oscillator behaves differently, as the energy becomes quantized, meaning it can only take discrete values. The energy levels are given by:

$$E_n = \hbar\omega \left(n + \frac{1}{2} \right)$$

Where $n = 0, 1, 2, \dots$, \hbar is the reduced Planck constant, and ω is the angular frequency. One of the most important results of this model is the existence of a non-zero minimum energy called the zero-point energy, given by:

$$E_0 = \hbar\omega \left(0 + \frac{1}{2} \right) \quad , \quad E_0 = \frac{1}{2} \hbar\omega$$

Which implies that the particle cannot be completely at rest even in its lowest energy state. Thus, the quantum harmonic oscillator highlights the fundamental difference between classical and quantum descriptions, where energy changes from being continuous in classical physics to being discrete and quantized in quantum mechanics.

Example:

Find the energy of the second excited state of a quantum harmonic oscillator.

Solution:

For $n=2$:

$$E_2 = \hbar\omega \left(2 + \frac{1}{2}\right)$$
$$E_2 = \frac{5}{2} \hbar\omega$$

Spectroscopy

Spectroscopy is a scientific technique used to study the interaction of electromagnetic radiation with matter by analyzing the light that is absorbed, emitted, or scattered. This is done through examining the spectrum of the substance, where each material possesses a unique spectral pattern like a fingerprint that allows it to be identified within the electromagnetic spectrum.

المطيافية (Spectroscopy) هي تقنية علمية تُستخدم لدراسة تفاعل الإشعاع الكهرومغناطيسي مع المادة من خلال تحليل الضوء الذي تمتصه أو تصدره أو تشتته، وذلك عبر دراسة الطيف (Spectrum) الخاص بها، حيث تمتلك كل مادة نمطاً طيفياً مميزاً يشبه بصمة يمكن من خلالها التعرف عليها ضمن الطيف الكهرومغناطيسي.

Types of Spectroscopy:

There are different types of spectroscopy based on the range of light and the type of transitions involved:

1-Absorption Spectroscopy:

is the study of how a substance absorbs specific wavelengths of light when electromagnetic radiation passes through it within the electromagnetic spectrum. It mainly includes UV-Vis, which involves electronic transitions, and IR, which involves molecular vibrations.

هو دراسة امتصاص المادة لاطوال موجية محددة من الضوء عند مرور الاشعاع الكهرومغناطيسي خلاله ضمن الطيف الكهرومغناطيسي. ويشمل نوعين UV-Vis الذي يسبب انتقال الالكترونات , و IR الذي يسبب اهتزاز الروابط الجزيئية.

2- Emission Spectroscopy:

Studies the light emitted by a substance after it has been energized, and is used to identify the elements present.

تدرس الضوء الذي تصدره المادة بعد اثارها بالطاقة ويستخدم لتحديد العناصر الموجودة فيها.

5- Nuclear Magnetic Resonance (NMR) Spectroscopy: it is based on the interaction of atomic nuclei with an external magnetic field, and it is mainly used to determine the chemical structure of organic compounds.

تعتمد على تفاعل انوية الذرات مع مجال مغناطيسي خارجي, وتستخدم بشكل اساسي لتحديد البنية الكيميائية للمركبات العضوية.

6- Raman Spectroscopy: it is based on the scattering of light by molecules, where a portion of the lights energy changes due to interactions with molecular.

تعتمد على تشتت الضوء بواسطة الجزيئات, حيث يتغير جزء من طاقة الضوء نتيجة التفاعلات مع الاهتزازات الجزيئية.

Applications of Spectroscopy:

- 1-Determining the chemical composition of materials
- 2-Studying the structure of atoms and molecules
- 3-Analyzing stars and outer space (as in astronomy)
- 4-Medical and environmental applications

The Zeeman Effect

Zeeman Effect is the splitting of a single spectral line into several lines when an atom is placed in an external magnetic field. A spectrum is formed when an electron inside an atom transitions from a higher energy level to a lower energy level.

When this happens, it emits a photon (light), and this photon appears in the spectrum as a spectral line.

-In the absence of a magnetic field:

The energy levels are not affected externally. Therefore, only one specific transition occurs. So, a single spectral line is observed.

-In the presence of an external magnetic field:

The electron is affected by the magnetic field. The energy levels split into several different sub-levels. As a result, multiple transitions occur. Therefore, several spectral lines appear instead of just one.

The energy E of the orbital magnetic dipole with magnetic field B , given by the equation:

$$E = -\mu_l B \cos \theta \quad \dots\dots\dots (1)$$

But the orbital magnetic dipole moment was found in equation:

$$\mu_l = -\frac{e}{2m_e} L$$

نعوض قيمة عزم ثنائي القطب المغناطيسي المداري μ_l في معادلة (1)

$$E = +\frac{e}{2m_e} LB \cos \theta$$

But $LB \cos \theta = B (L \cos \theta)$

Also we have $L \cos \theta = L_z$

$$E = +\frac{e}{2m_e} L_z B$$

Finally, substituting for $L_z = m_l \hbar$ we get:

$$E = +m_l \frac{e\hbar}{2m_e} B$$

$$E = +m_l \mu_B B$$

μ_B represent **Bohr Magneton** :It is the amount of magnetic dipole moment that an electron can have inside an atom due to its motion.