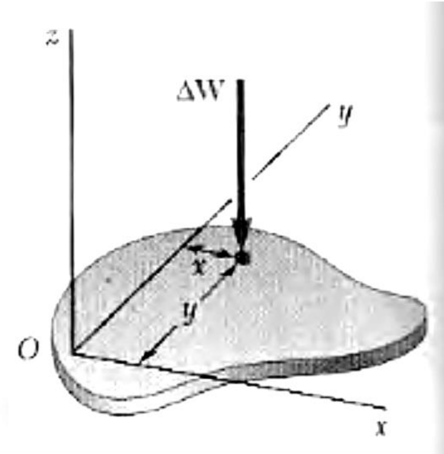
Centroids and Center of Gravity:

Center of gravity : is the point through which the line of action of the weight always passes.
Locating the C.g of a body = determining the point through which the resultant force of gravity of the body acts.

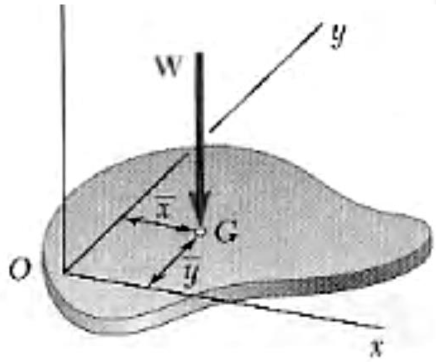


A plate with irregular section is divided into small elements having weights w_1 , w_2 ,w_3 , ….. w_n so as

$$\sum_{1}^{n} w_i = W$$ (total weight of the plate)

$$\sum_{1}^{n} M_{y-y} : \; W\bar{x} = w_1x_1 + w_2x_2 + \dots\dots\dots = \Sigma\, w_ix_i$$

$$\sum_{1}^{n} M_{x-x} : \; W\bar{y} = w_1y_1 + w_2y_2 + \dots\dots\dots = \Sigma\, w_iy_i$$



Decreasing the size of each element at the limit we can write :

W = ∫ dw

$\bar{x}$ **W = ∫ x dw**

$\bar{y}$ **W = ∫ y dw**

Note : c.g may not located on the body. (as in arc)

Centroids of Areas and Lines:

W = V *ρ = A * t * ρ where **V**: volume **ρ** : density
A : area **t** : thickness

A t ρ $\bar{x}$ = a_1 t ρ x_1 + a_2 t ρ x_2 + ……….= t ρ Σ a x

⇒ A $\bar{x}$ = Σ a x & A $\bar{y}$ = Σ a y

$$\bar{x} = \frac{\Sigma a_i x_i}{A} \quad ; \quad \bar{y} = \frac{\Sigma a_i y_i}{A}$$

$\bar{x}$ **A = ∫ x dA** ; $\bar{y}$ **A = ∫ y dA**

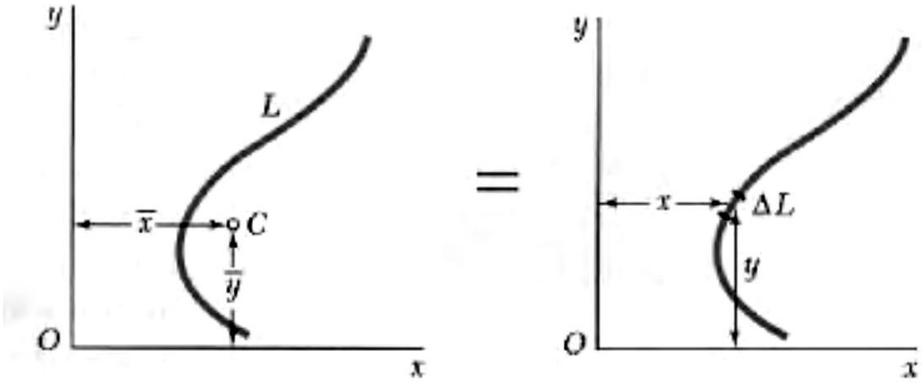
The point ($\bar{x}$, $\bar{y}$)known as the centroid "**c**" of the area "**A**" of the homogenous plate.
∫ x dA: is the 1st moment of area "**A**" with respect to the **y-axis** & **∫ y dA:** is the 1st moment of area "**A**" with respect to the **x-axis**.
Note : The center of area of a plane (circle , square , …..)is known as centroid .

For a Homogenous Wire :

W = a * l * ρ

$$\bar{x} = \frac{\Sigma l_i x_i}{l}$$

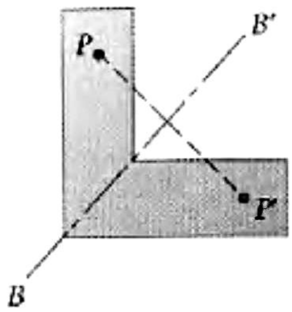
$$\bar{y} = \frac{\Sigma l_i y_i}{l}$$



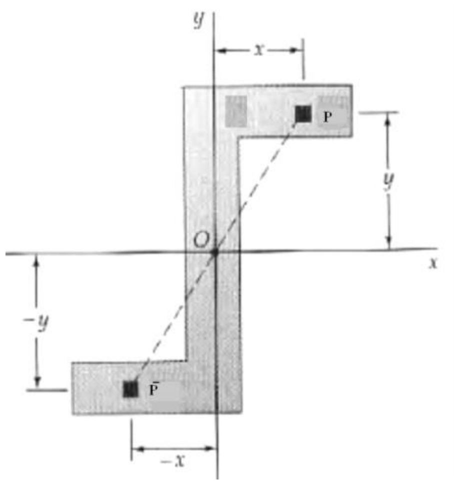
* Importance of Centroids and Moments of Area :

- Strength
  - Centroid of area (uniform stress distribution)
  - Centroidal axis → neutral axis.(bending stress)
  - Centroidal axis → moment of inertia
  - Moment of area
    - shearing stress in beams
    - deflection of beams

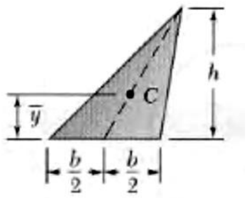
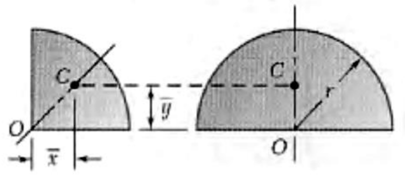
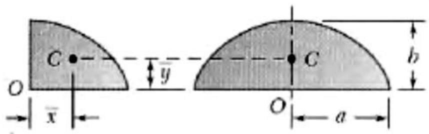
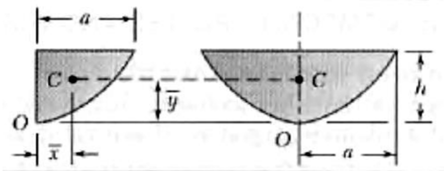
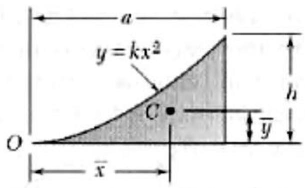
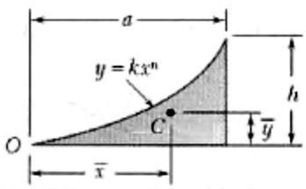
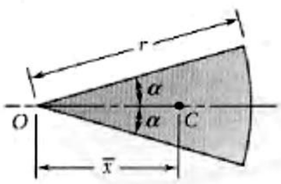
* An area "A" is said to be symmetrical about an axis $B\bar{B}$ if that every point "P" of the area corresponds a point " $\bar{P}$ " of the same area such that the line $P\bar{P}$ is ⊥ to $B\bar{B}$ & is divided in to two equal parts by that axis.

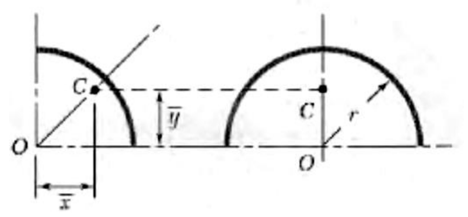
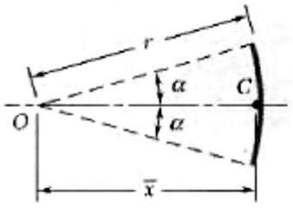


* An area "A" is said to be symmetrical about an "O" if that every point "P" of the area corresponds a point " $\bar{P}$ " of the same area such that the line $P\bar{P}$ is divided in to two equal parts by "O".



Centroids of common shapes of areas

Shape		$\bar{x}$	$\bar{y}$	A
Triangular are		-	$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-Circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-Elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r\sin\alpha}{3\alpha}$	0	αr^2

<table>
<tr><th>Shape</th><th></th><th>$\bar{x}$</th><th>$\bar{y}$</th><th>A</th></tr>
<tr><td>Quarter-Circular arc</td><td rowspan="2">
</td><td>$\frac{2r}{\pi}$</td><td>$\frac{2r}{\pi}$</td><td>$\frac{\pi r}{2}$</td></tr>
<tr><td>Semicircular arc</td><td>0</td><td>$\frac{2r}{\pi}$</td><td>πr</td></tr>
<tr><td>Arc of Circle</td><td>
</td><td>$\frac{r\sin\alpha}{\alpha}$</td><td>0</td><td>$2\alpha r$</td></tr>
</table>