Introduction

Natural numbers={1,2,3,…..},Whole numbers={0,1,2,3,….}

Integers numbers$\{0, \mp 1, \mp 2, \mp 3, \dots\}$,

Real numbers$\{0,\mp 1, \mp 2, \mp 3, \frac{1}{2}, \sqrt[3]{25}\}$

Rational numbers=$\{\frac{3}{2}, \frac{8}{4}, \dots\dots)$

Sets:-

Empty set $A = \{\emptyset\}$

Equal sets as: A={5,6,9,0) B={0,9,5,6} A=B

Intervals:

Closed interval [a,b],$a \leq x \leq$b,*Open interval* *(a,b) ,a<x<b*

Semi- open interval (a,b], a<x≤b

Semi closed interval [a,b),a≤x<b

The equation of line and slope: it is in the form $Ax + By = C$ (A and B not both zero)

The slope of the line through $P_1(x_1, y_1)$ and $P_2(X_2, Y_2)$ $X_1 \neq X_2$, is

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{\Delta y}{\Delta x}$$

m = tan φ($\varphi$is the angle of inclination).

a-Vertical lines have no slope.
b-Horizontal lines have slope 0.
c-For lines that are neither horizontal nor vertical it is may be :

a) they are parallel $m_2 = m_1$
b) they are perpendicular , $m_2=-1/m_1$.

Equations for Lines

$X=a$	Vertical line through *(a, b)*
$Y=b$	Horizontal line through *(a, b)*
$Y=mx+b$	Slope–intercept equation
$y-y_1=m(x-x_1)$	Point–slope equation

Ex :find the equation for the line passes through the two points:(-2,1) And(2,-2)?

m=$\frac{\Delta y}{\Delta x}=\frac{-2-1}{2-(-2)}$=$\frac{-3}{4}$

y-y1=m(x-x1)   at (-2,1) , y-1=$\frac{-3}{4}$(x+2)   ,y= $\frac{-3}{4}$x-$\frac{1}{2}$

<u>**Functions:-**</u>A function is a rule that associated an unique output with each input. If the input is denoted by x then the output is denoted by f(x).

The input value of x $\rightarrow$ domain

The output value of y $\rightarrow$ range

Families of functions:

1. lines as: f(x)=c,f(x)=mx+b.
2. power function as $y=x^n$:$y=x^3$,$y=x^4$,$y=x^{-7}$.
3. Rational functions: y=$\frac{x^2+2x}{x^2-1}; y=\frac{x^2-8}{x-6}; y=\frac{3}{x^4+1}$.
4. Algebraic function: y=$\sqrt{x^2-4}$;y=$9\sqrt[3]{x}(2+x)$.
5. Trigonometric functions: y=sinx,y= tanhx.
6. Exponential and logarithm functions: y=e^{x-6};y=log_6^{4x+1}.

Find domain and range of the following functions:
1.$y=x^2$

Domain={x:x all real numbers}

Range={y:y≥0} or {all real numbers except negative numbers}

2.y=$\frac{1}{3-x}$ domain={all real numbers except 3}

range={y:y all real numbers except 0}

Inequality: solve a linear inequalities:

1. 8-3x≤ 20 ; 8-8-3x≤20-8; -3x≤12

x≥-4

2. 9x-4>5x+2; 9x$-4+4>$5x+2+4

9x$-6>$5x+6-6 ; 9x-5x>6 ; x$>\frac{3}{2}$

3. $\frac{2x}{3}+12<\frac{x}{6}$+18; ($\frac{2x}{3}+12<\frac{x}{6}$+18)*6 ; 4x+72$<x+108$

$x<12$

4. -7$\leq 5x-2<$8 (this is double inequality)

-7+2$\leq 5x-2+2<8+2$; -5 ≤5x<10 ; -1$\leq x<2$

Or solve as double inequality:-

-1$\leq x$ **&** **x**<2

Absolute value and properties:-

/x/=$\sqrt{x^2}$=$\begin{Bmatrix} +x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{Bmatrix}$

1./x/≥ 0 2./-x/=/x/=x 3./x-a/=/a-x/ 4./x.y/=/x/*/y/

5./$\frac{x}{y}$/$=\frac{/x}{/y}$ y≠0 6./x+y/≤/x/+/y/

7./x/≤a -a≤x≤+a 8./x-y/≥/x/-/y/ 9.a-c≤x≤a+c / x-a/≤c

Ex: solve the equation:/2x-3/=7?

2x-3=7 ► x=5 or 2x-3=-7 ► x=-2

This equation has two solutions at x=5 and x=-2.

Solving inequalities involving absolute value:

Let x be a variable or an algebraic expression and let be a real number such that a> 0:

1. The solution of /x/$< a$ are values of x that lie between –a and a that is :

/x/$< a \; if \; and \; only \; if \quad -a < x < +a$

2. The solution of /x/> a $are \; all \; values \; of \; x \; that \; are \; less \; than$

$$-a \; or \; greater \; than \; a \; that \; is$$

/x/>a if and only if x<-a <u>or</u> x>+a

This rules are also valid if < is replaced by≤ and > is replaced by ≥ .

3-Solving an equation involving two absolute values:

/ax+b/=/cx+d/ by forming two linear equations

ax+b=cx+d ax+b=-(cx+d)

Ex: solve an absolute value in quality /x-5/<2 ?

-2<x-5<+2 ► +3<x<7 and check it .

Ex:solve /3x-4/≥ 5?

3x-4 ≤-5 or 3x-4≥+5

3x≤ -1 3x ≥ 9

x $\leq -\frac{1}{3}$ x≥ 3

the solution is the set $(\infty, \frac{1}{3}] \cup [3, \infty)$.

Ex: solve 1. /3x-4/=/7x-16/

3x-4=7x-16 or 3x-4=-7x+16

X=3 x=2

Solutions are 3 and 2.

2./x+5/=/x+11/

X+5=x+11 or x+5=-x-11

0≠6 2x=-16

X=-8

This equation has only one solution at x=-8

Complex numbers:-

an expression of the form x+yi, is called a complex number. It is usually denoted by z=x+yi ; x is called the real part and y the imaginary part of z and may be denoted by Re(z) and Im(z) respectively, as: 2+3i,1/2+5i.

Basic properties:

Equality: a+bi=c+di iof and only if a=c and b=d

Addition : (a+bi)+ (c+di)= (a+c)+ (b+d)i

Multiplication : (a+bi)* (c+di)=(ac-bd)+ (ad+bc)i

Ex: find 1.(2-3i)+(2+i)=2-3i+2+i=4-2i

2.(7-3i)-(6+2i)=7-3i-6-2i=1-5i

$=12+4i-18i-6i^2$

$=12+4i-18i+6$

$=18-14i$

Conjugate of complex number: the conjugate of a complex number z=x+yi is denoted by $\bar{z}$ and is defined as z=x-yi

Let z=5-3i,$then\ \bar{z} = 5 + 3i$

Let z=3i=0+3i then $\bar{z} = 0 - 3i = -3i$ (called purely imaginary)

Properties of conjugate: for any three complex numbers z,z1,z2 ,we have: 1.$\bar{\bar{z}}$ =z 2.z+$\bar{z}$=2Re(z) 3.z-$\bar{z}$=2i Im(z) 4.z$\bar{z}$=$[Re(z)]^2+[Im(z)]^2$

5.z=$\bar{z}$ if and only if z is purely real

6. $\bar{z}$=-z if and only if z is purely imaginary

7.$\overline{z1 + z2} = \overline{z1} + \overline{z2}$ 8.$\overline{z1z2} = \overline{z1}\ \overline{z2}$ 9.$\overline{\left(\frac{z1}{z2}\right)} = \frac{\overline{z1}}{\overline{z2}}$,where $\overline{z2} \neq 0$

***To write complex number $\frac{a+bi}{c+di}$ in the form A+Bi** we should multiply the numerator and the denominator by the conjugate of the denominator.

***modulus of complex number:** modulus of complex number z=a+bi is denoted by mod(z)or/z/ and is defined as:

/z/=$\sqrt{a^2+b^2}$ or called absolute value of z ,where /z/≥0

Example: express the following in the form a+bi :

1.(5i)$\left(-\frac{3}{5}i\right)$ =$5 * -\frac{3}{5} * i^2$= -3$i^2 = 3$

2.i^9+bi^{19}= $i.i^8+i.i^{18}$ = $i.(i^2)^4+i.(i^2)^9$ = $i.(-1)^4+i.(-1)^9$ =i - i=0

3.$\left(\frac{1}{5} + \frac{2}{5}i\right) - \left(4 + \frac{5}{2}\right)i = \left(\frac{1}{5} + \frac{2}{5}i\right) + \left(-4 - \frac{5}{2}\right)i = \left(\frac{1}{5} - 4 + \frac{2}{5}i - \frac{5}{2}i\right)$

$= -\frac{19}{5} - \frac{21}{10}i$

4.$\frac{(2-8i)(7+8i)}{1+i} = \frac{14+16i-56i-64i^2}{1+i} = \frac{14-40i+64}{1+i} = \frac{78-40i}{1+i} * \frac{1-i}{1-i} =$

$\frac{78-78i-40i+40i^2}{1-i^2} = \frac{28-118i}{2}$**=19-59i**

5. $\left(\frac{4i^3+1}{2i+1}\right)^2 = \left(\frac{-4i-1}{2i+1}\right)^2 = \frac{(1+4i)^2}{(1+2i)^2} = \frac{1+16i^2+8i}{1+4i^2+4i} = \frac{1-16+8i}{3-4+4i} = \frac{-15+8i}{-3+4i} = \frac{15-8i}{3-4i} *$
$\frac{3+4i}{3+4i} = \frac{77}{25} + \frac{36}{25}i$

Ex: find x and y if $(3x-2yi)(2+i)^2=10(1+i)$

$(3x-2yi)(4+4i+i^2)=10+10i$

$(3x-2yi)(3+4i)=10+10i$ ►$(9x-6yi+12xi-8yi^2)=10+10i$

$9x+8y+(12x-6y)i=10+10i$

equating real and imaginary parts, we get

$9x+8y=10$ …….(1) and $12x-6y=10$……(2)

By solving (1)&(2):

$x=14/5,\quad y=1/5$

Limits:- $\lim_{x\to c} f(x) = l; l = cons.$

Ex: find the limit : 1.lim (x3-2x2+4x+8)=16

2.lim 5(2x-1)=-5

Properties of Limits

If $\lim_{x\to c} f_1(x) = L_1$ and $\lim_{x\to c} f_2(x) = L_2$, then

1. *Sum Rule:* $\lim [f_1(x) + f_2(x)] = L_1 + L_2$
2. *Difference Rule:* $\lim [f_1(x) - f_2(x)] = L_1 - L_2$
3. *Product Rule:* $\lim f_1(x) \cdot f_2(x) = L_1 \cdot L_2$
4. *Constant Multiple Rule:* $\lim k \cdot f_2(x) = k \cdot L_2$ (any numbe
5. *Quotient Rule:* $\lim \frac{f_1(x)}{f_2(x)} = \frac{L_1}{L_2}$ if $L_2 \neq 0$.

The limits are all taken as $x \to c$, and L_1 and L_2 are real numbers.

Ex:find lim $\frac{x^3-8}{x^2-4} = \frac{0}{0}$; lim$\frac{(x-2)(x^2+2x+4)}{(x+2)(x-2)} = \frac{12}{4} = 3$

<u>Relationship between one sided and two sided limits:</u>

A function f(x) has a limit as x approaches c if and only if the right-hand and left-hand limits at c exist and are equal .in symbols

Lim f(x)=L ↔ lim f(x)=L and lim f(x)=L

Ex: find the limit if exist for the function

f(x)= $\begin{bmatrix} x^2+2 & x \geq 2 \\ 2x+2 & x < 2 \end{bmatrix}$

limf(x)=4+2=6 ;lim f(x)=2*2+2=6

this function has limit =6 at x=2

ex: find the limit if exist at x=3

f(x)= $\begin{bmatrix} x^2+1 & x < 3 \\ x-2 & x \geq 3 \end{bmatrix}$

lim (x^2+1)=10 lim(x-2)=1

∴ there is no limit at x=3 because lim(x2+1)≠lim(x-1)

Ex: find the limit if exist lim $\frac{/x/}{x}$

Lim $\frac{\mp x}{x}$; lim $\frac{+x}{x}$ =1 ; lim$\frac{-x}{x}$=-1

"There is no limit for this function"

<u>Limits of rational functions as x→∓ ∞:-</u>

a. Lim $\frac{f(x)}{g(x)}$ is 0 if deg(f)< g(x)
b. Lim $\frac{f(x)}{g(x)}$ is finite if deg(f)=deg(g)
c. Lim $\frac{f(x)}{g(x)}$ is infinite if degf()> deg(g)

Ex: find limit of the following:

1. lim $\frac{5x+2}{2x^3-1}$ 2. lim $\frac{-x}{7x+4}$ 3.lim $\frac{3x^3-6x}{4x-8}$

1. lim $\frac{5x+2}{2x^3}$ (divide the numerator and denominator by the highest power of x in the denominator in this case)

Lim $\frac{\frac{5x}{x^3}+\frac{2}{x^3}}{\frac{2x^3}{x^3}-\frac{1}{x^3}}$ =0

2. lim $\frac{-x}{7x+4}$ (divide the numerator and denominator by the highest power of x in denominator in this case)

Lim $\frac{-1}{7+\frac{4}{x}}$ =$\frac{-1}{7}$

3. lim $\frac{3x^3-6x}{4x-8}$ (divide the numerator and denominator by the highest power of x in denominator in this case)

Lim $\frac{\frac{3x^3}{x}-\frac{6x}{x}}{\frac{4x}{x}-\frac{8}{x}}$ =$\frac{3x^2-6}{4-0} = \infty$

*continuous at an interior point:

A function y= f(x) is continuous at an interior point c of its domain if lim f(x)=f(c)

The function is continuous if

1.f(c) is exist 2.lim f(x) is exist 3.lim f(c)=f(c)

Ex: if the following function continuous at x=-3

f(x)= $\begin{cases} \frac{x^2-9}{x+3} & x \neq -3 \\ -6 & x = -3 \end{cases}$

1. F(a)=f(-3)=-6
2. Lim f(x)= lim $\frac{x^2-9}{x+3}$=lim $\frac{(x+3)(x-3)}{(x+3)} = \lim(x-3) = -6$
3. $\therefore limf(x) = f(-3)$
   This function continuous at x=-3
   Ex: is the following function contiguous at x=1:
   F(x)=$\frac{1}{x-1}$ $x \neq 1$
   F(1)=∞ not exist or not defined
   Lim $\frac{1}{x-1} = \infty$ this function is discontinuous