5. Domain and Range

If the value of one variable y is completely determined by the value of x (that is, y depends on the value of x), we say that y is a function of x. Often the value of y is given by a **rule** or **formula** that says how to calculate it from the variable x. For instance, the equation $A = \pi r^2$ is a rule that calculates the area A of a circle from its radius r.

A symbolic way to say y is a function of x is by writing:

$$y = f(x) \qquad (y \text{ is a function of } x)$$

In this notation:

The symbol **f** represents the function.
The letter **x**, called **the independent variable**, represents the input value of **f**.
y , **the dependent variable**, represents the corresponding output value of **f** at **x**.

Domain(D_f) represents values of **(x)**;

Domain(R_f) represents values of **(y)**;

For real-valued **domains and ranges**, *the following points should be satisfied*:

1. We cannot divide by zero. The *denominator* cannot be *zero*. $(\frac{a}{0})$
2. Any value under the square root cannot be negative

- To find the Domain (Df):
1. Find (y) in terms of (x);
2. Avoid: (a) division by zero; (b) root value (complex number $(\sqrt{-})$

- To find the Range:
1. Find (x) in terms of y;
2. Avoid: (a) division by zero; (b) root value (complex number $(\sqrt{-})$

Example 1: Find the domain and range of $y = x^2 + 1$

Solution: D_f; $-\infty \leq x \leq \infty$ or;;; $D_f : R$

R_f; $x^2 = y - 1$

$x = \pm\sqrt{y-1}$

$y - 1 \geq 0$;; $y \geq 1$

Example 2: Find the domain and range of $y = \sqrt{x+1}$

Solution: D_f; $x + 1 \geq 0$;;; $x \geq -1$

R_f; $y \geq 0$ $\sqrt{+} = +$

Example 3: Find the domain and range of $y = 1 + \sqrt{x}$

Solution: D_f; $x \geq 0$

R_f; $\min\sqrt{0} = 0$;;; $y = 1 + 0 = 1$;;; $y \geq 1$

Example 4: Find the domain and range of $(x-2)^2 + (y+1)^2 = 9$

Solution:

$$(y+1)^2 = 9 - (x-2)^2$$
$$(y+1) = \pm\sqrt{9-(x-2)^2}$$
$$y = -1 \pm \sqrt{9-(x-2)^2}$$

$9-(x-2)^2 \leq 0$;;; $(x-2)^2 \leq 9$;;; $-3 \leq (x-2) \leq 3$

D_f; $-1 \leq x \leq 5$

R_f;

$$(x-2)^2 = 9-(y+1)^2$$
$$(x-2) = \pm\sqrt{9-(y+1)^2}$$
$$x = 2 \pm \sqrt{9-(y+1)^2}$$

$9-(y+1)^2 \geq 0$;; $(y+1)^2 \leq 9$;;; $-3 \leq (y+1) \leq 3$

R_f; $-4 \leq y \leq 2$

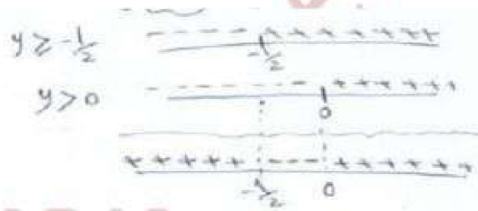
Example 5: Find the domain and range of $y = \frac{1}{x-4} - \frac{1}{x+4}$

Solution: D_f; $x \neq 4 \ \neq and\ x \neq -4$

$$y = \frac{(x+4)-(x-4)}{(x-4)\times(x+4)} = \frac{8}{x^2-16}$$

$$y(x^2 - 16) = 8$$

$yx^2 - 16\,y = 8$;;; $yx^2 = 8 + 16\,y$ $x = \pm\sqrt{\frac{8+16\,y}{y}}$



$8 + 16\,y \geq 0$;;; $y \geq -\frac{1}{2}$;;; $y > 0$

R_f; $y \leq -\frac{1}{2}$ $\cup$ $y > 0$

Example 6: Find the domain of $y = \sqrt{x-1} + \sqrt{9-x^2}$

Solution: $x - 1 \geq 0$;;; $x \geq 1$

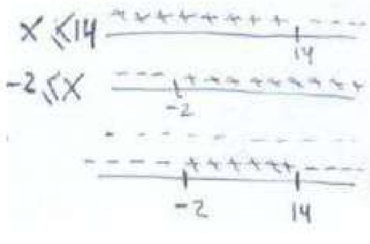
$9 - x^2 \geq 0$;; $x^2 \leq 9$;; $-3 \leq x \leq 3$

D_f; $1 \leq x \leq 3$

Example 7: Find the domain and range of $y = \sqrt{4 - \sqrt{x+2}}$

Solution: $0 \leq 4 - \sqrt{x+2}$;;; $\sqrt{x+2} \leq 4$;; $x + 2 \leq 16$;;; $x \leq 14$

For $\sqrt{x+2}$; $0 \leq x + 2$;;; $-2 \leq x$



D_f: $-2 \leq x \leq 14$

For y min $\sqrt{x+2} = 0$;; $y = \sqrt{4-0}$

Max. $\sqrt{x+2} = \sqrt{14+2} = 4$;; $y = \sqrt{4-4} = 0$

R_f: $0 \leq y \leq 2$