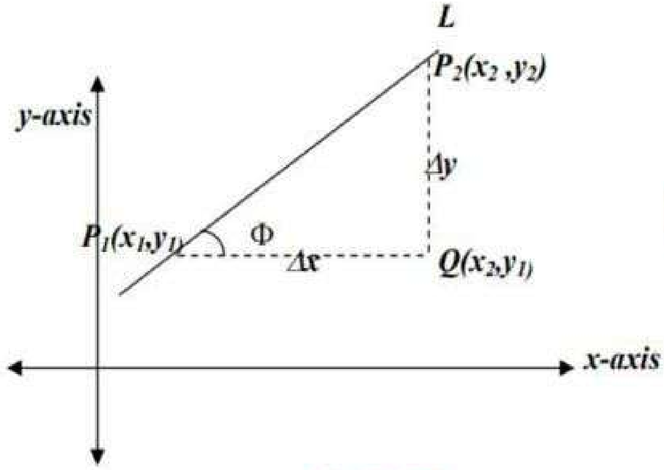
1. Equations of the Lines

Slope of the line:

$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$ where $\Delta x \neq 0$

$m = \tan\theta$



Equations of the line:

$y = mx + b$

$y - y_1 = m(x - x_1)$

where (x_1, y_1) any point on the line

Notes:

- A line that goes uphill as x increases has a positive slope .
- A line that goes downhill as x increases has a negative slope .
- A ***horizontal line*** has zero slope; because $\Delta y = 0$.

  [y = b, horizontal line]
- The slope of a ***vertical line*** is undefined; because $\Delta x = 0$.

  [x = a, vertical line]
- If two lines are parallel then: $m_1 = m_2$
- If two lines are perpendicular then: $m_1 = \frac{-1}{m_2}$
- Distance bewteen two points on a line (d) :
- $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Example1: Find the equation of the line passes through the points $P_1(4, 6)$ and $P_2(6, 10)$.

Solution:

$m = \frac{\Delta y}{\Delta x} = \frac{10-6}{6-4} = 2$

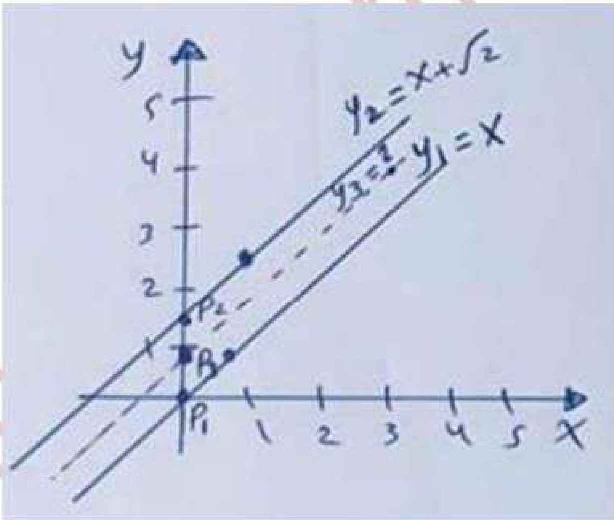
$y - y_1 = m\,(x - x_1)$

$y - 6 = 2\,(x - 4)$; $y - 6 = 2x - 8$; $\therefore y = 2x - 2$

$y - 10 = 2\,(x - 6)$; $y - 10 = 2x - 12$; $\therefore y = 2x - 2$

Example2: Two lines are parallel $[y_1 = x_1 \quad and \quad y_2 = x_2 + \sqrt{2}]$. Find the equation of the line that passes between these two lines (*midway*).

Solution:



- **Line1**

x_1	$y_1 = x_1$	Points
0	0	(0,0)
1	1	(1,1)

- **Line 2**

x_2	$y_2 = x_2 + \sqrt{2}$	Points
0	$\sqrt{2} = 1.41$	(0,1.41)
1	$1 + \sqrt{2} = 2.41$	(1,2.41)

- The two lines are parallel then:
  $m_1 = m_2 = m_3$
- P_1 (0, 0) and P_2 (0, 1.41),
- then the third point (which is on the third line) is:

$P_3 = \left(\frac{0-0}{2}, \frac{1.41-0}{2}\right) = (0, 0.7)$

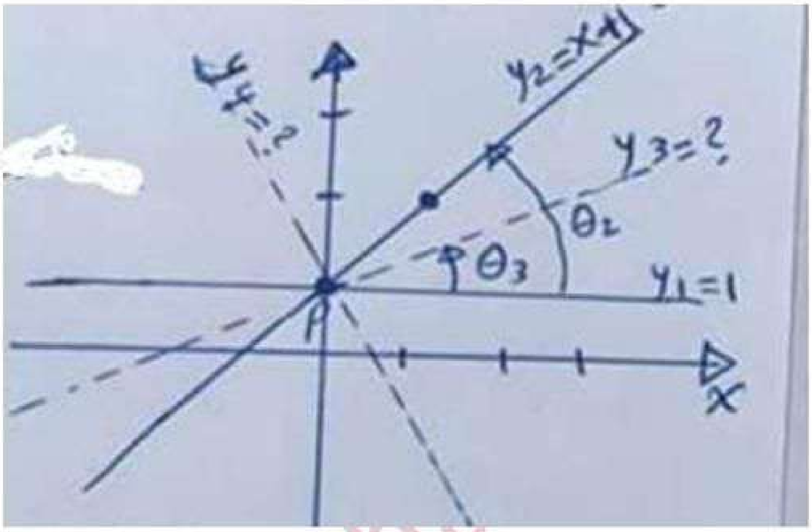
$y - y_1 = m\,(x - x_1)$;;; $y - 0.7 = 1\,(x - 0)$; $y = x + 0.7$; $\therefore\ y_3 = x_3 + 0.7$

Example3: Two lines are parallel $[y_1 = 1 \quad and \quad y_2 = x_2 + 1]$. Find the equation of the line that passes through the intersection point and Halfling the angle between these lines.

Solution:

Line 1: $m_1=0$ ($y_1= 1$; horizontal line) $\therefore \theta_1=0$

Line2:



x_2	$y_2=x_2+1$	Points
0	1	(0,1)
1	2	(1,2)

$m_2=1$ ($y_2= x_2 +1$)

$m_2 = \tan \theta_2$

$1= \tan \theta_2$

$\theta_2 = \tan^{-1}(1)$

$\therefore \theta_2 = 45°$

$$\theta_3 = \frac{\theta_2 - \theta_1}{2} = \frac{45° - 0}{2} = 22.5°$$

$m_3 = \tan \theta_3$

$m_3 = \tan 22.4° = 0.414$ and the point of intersect (0, 1)

$y - y_1 = m (x - x_1)$

$y - 1= 0.414 (x - 0)$; $\therefore y_3 = 0.414\, x_3 + 1$

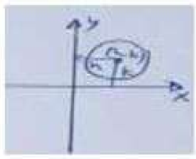
$\theta_4 = 22.5° + 90° = 112.5°$

$m_4 = \tan 112.5° = -2.414$ and the point of intersect (0, 1)

$y - y_1 = m (x - x_1)$;;;;; $y - 1= -2.414 (x - 0)$; $\therefore y_4 = -2.414\, x_4 + 1$

Equation of a circle:

1. $(x - h)^2 + (y - k)^2 = R^2$; where (h, k) the center of the circle



2. $x^2 + y^2 = R^2$; $(h, k) = (0,0)$

Example 4: Find the center and radius of the following circle:

$x^2 + y^2 - 2x - 4y - 11 = 0$

Solution:

$$x^2 + y^2 - 2x - 4y - 11 = 0$$

$$x^2 - 2x + 1 - 1 + y^2 - 4y + 4 - 4 = 11$$

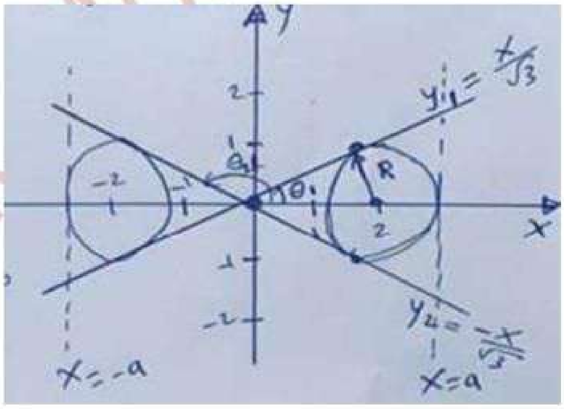
$$(x-1)^2 - 1 + (y-2)^2 - 4 = 11$$

$$(x-1)^2 + (y-2)^2 = 16$$

∴ Center of the circle (1,2); Radius of the circle 4

Example 5: Three lines $\left[y_1 = \frac{x_1}{\sqrt{3}};\ y_2 = -\frac{x_2}{\sqrt{3}};\ x_3 = a\right]$ are tangent to a circle of a radius equal to (R=1). Find the value of a.

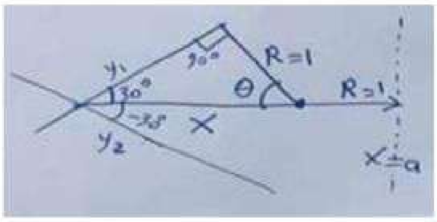
Solution:



Line1: $m_1 = \frac{1}{\sqrt{3}} = \tan\theta_1$

$\tan\theta_1 = \frac{1}{\sqrt{3}}$; $\therefore \theta_1 = 30°$

x_1	$y_1 = \frac{x_1}{\sqrt{3}}$	Points
0	0	(0,0)
$\sqrt{3}$	1	$(\sqrt{3},1)$



Line 2: $m_2 = -\frac{1}{\sqrt{3}} = \tan\theta_2$

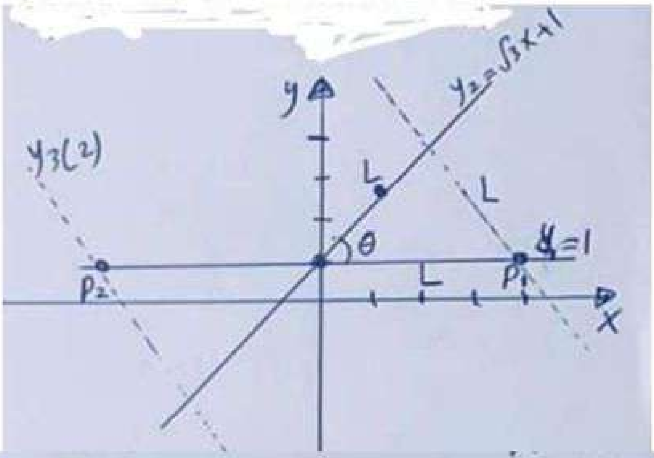
$\tan\theta_2 = -\frac{1}{\sqrt{3}}$; $\therefore \theta_2 = -30°$

x_2	$y_2 = -\frac{x_2}{\sqrt{3}}$	Points
0	0	(0,0)
$\sqrt{3}$	- 1	$(\sqrt{3},-1)$

$\sin 30° = \frac{R}{x}$ $\quad \frac{1}{2} = \frac{1}{x}$ $\quad \therefore \quad x = 2$

X+R = a ; 2+1=3 $\quad$ ∴ a =3

Example 6: Three lines $\left[y_1 = 1;\ y_2 = \sqrt{3}x_2 + 1;\ \ and\ y_3 = \mathrm{m}x_3 + \mathrm{b}\right]$ are intersect to form a triangle with equal sides length. Find the equation of the third line, if the area of the triangle equal to $\left(\frac{3\sqrt{3}}{4}\right)\ unit^2$.



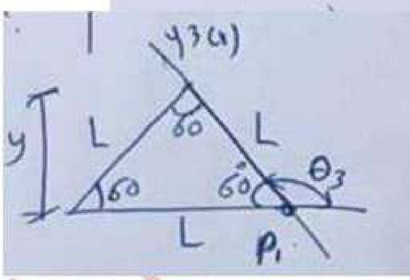
Solution:

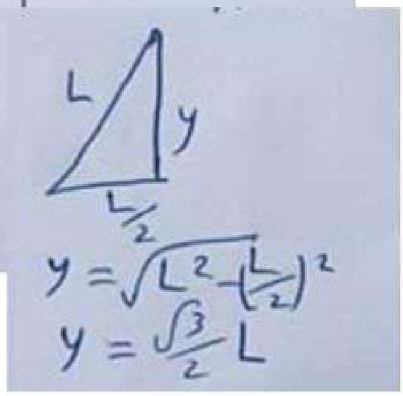
- **Line 2**

x_2	$y_2 = \sqrt{3x_2} + 1$	Points
0	1	(0,1)
1	2.7	(1,2.7)

$m_1 = 0\ \ ;\ \ \theta_1 = 0$

$m_2 = \sqrt{3} = \tan\theta_2$





$\tan\theta_2 = \sqrt{3}\ \ ;\ \therefore \theta_2 = 60°$

$\therefore \theta_3 = 180° - 60° = 120°$

$m_3 = \tan 120° = -\sqrt{3}$

$A = \frac{1}{2} L\, y$

$A = \frac{1}{2} L \frac{\sqrt{3}}{2} L \qquad ;\ A = \frac{\sqrt{3}}{4} L^2$

$\frac{3\sqrt{3}}{4} = \frac{\sqrt{3}}{4} L^2 \qquad \therefore L = \sqrt{3}$

1. $P_1(\sqrt{3}, 1)\ \ and\ \ m_3 = -\sqrt{3}$

1. $y - y_1 = m(x - x_1)$

$y - 1 = -\sqrt{3}(x - \sqrt{3})\ ;\ y = -\sqrt{3}x + 3 + 1;\ \ \therefore y_3 = \sqrt{3}x_3 + 4$

2. $P_2(-\sqrt{3}, 1)\ \ and\ \ m_3 = -\sqrt{3} \qquad y - y_1 = m(x - x_1)\,;$

$y - 1 = -\sqrt{3}(x + \sqrt{3})\ ;\ y = -\sqrt{3}x - 3 + 1;\ \ \therefore y_3 = -\sqrt{3}x_3 - 2$