Example:- The probability passes Ali of math exam is 4/5 and the probability he passes the chemistry exam is 5/6. If the probability that he passes both exams is ¾, find the probability that he will pass at least one exam.

Sol: - let M = Ali pass Math exam,

C = Ali pass Chemistry exam

P (Ali pass at least one exam) =P (M∪C)

= P (M) + P(C) - P (M∩C)

= 4/5+5/6-3/4=53/60.

Example:- If the probability of success of Mohamed in one of the tests is equivalent to 0.6 and the probability of success of Mohammed and Ahmed together in this test is equal to 0.1. Find the probability of the success of Mohammed and the failure of Ahmed**.**

A= {success of Mohammed}

B= {success of Ahmed}

A∩B= {Success of Mohammed and Ahmed}

B^c= {failure of Ahmed}

A∩B^c= {success of Mohammed and failure of Ahmed}

P (A) =0.6, P (A∩B) =0.1

P (A∩B^c) =P (A)-P (A∩B) =0.6-0.1=0.5

C= {success of Mohammed}, P (C) =0.25

D= {success of Ahmed}

D^c= {failure of Ahmed}, P (D^c)= 0.3

C∩D= {Success of Mohammed and Ahmed}

P (C∩D) =0.1

Find Probability of success at least one of them.

Find Probability of failures at least one of them.?

Find Probability of failures Mohamed and Ahmed .?

C∪D= {Success at least one of them}

= {Success of Mohammed or success of Ahmed}

P(C∪D)= P(C)+P(D)-P(C∩D) =0.25+0.7-0.1 =0.85

1-P (D^c) =1-0.3=0.7

Example:- If the Probability that a blood type one blood donor is of type A is 0.35 and the probability it infected blood pressure is 0.15. And the probability donor has been infected with blood pressure or the group of his blood type A is 0.40.

Find the probability

1. The donor's Infected blood by pressure and its blood group type A.
2. The donor is not infected with blood pressure.
3. The blood type of the Donor is not A.
4. The blood type of the Donor is not A and it is infected blood pressure.
5. The blood type of the Donor is not A and is not infected with blood pressure.
6. The blood type of the Donor is not A or is not infected with blood pressure..

Sol

B = { The donor's infected by blood pressure}

A = { The blood type of the donor is A }

1- P(A∪B) =0.40 , P(B)=0.15 , P(A) = 0.35

P (A∩B) = P (A) + P (B) – P (A∪B)

= 0.35+0.15-0.40= 0.10

2- P(B^c)= 1- P(B) =1-0.15

=0.85.

NON-INDEPENDENT EVENTS

If A and B are non-independent events.

P (B|A) =P (A ∩ B)/P (A)
P (A ∩ B) = P (A) ▪ P (B|A)

Examples:- A die is rolled and recorded the number shown. if the number is less than 4, what is the probability that it is odd?

Sol: - let A= {number is odd} and B={less than 4}
A ∩ B= {1, 3}, P (A ∩ B) =2/6=1/3
P (B) =3/6=1/2
P (A|B) = P (A ∩B)/P (B)
$= \frac{1/3}{1/2} = \frac{2}{3}$.

Now, Are A and B independent, mutually exclusive, or complementary? Why?

Examples:- A die is rolled and the number showing recorded. Event A is the number less than or equal 4, B is odd? Find p (A∪B).A={1,2,3,4} , B={1,3,5}
p (A∪B)

Exercises

1. If the result of the experiment is getting on one of the following events and probability

P(F_1)=.15, P(F_2)=.33 , P(F_3)=.25, P(F_4)=.2,

And suppose that A={(F_1, F_2)} , B={(F_3)}, C={(F_1, F_3, F_4)}.

Find p(A),p(B),p(C),p(A∩B), p(C∩B), p(A|B) , p(A|C), p(B|B), p(B|A).

2. The box contains 2 black, 8 white, and 1 red ball. If the ball is pulled what's the probability of the ball being black or white? (addition law).

3. Two boxes, the first contains 6 white and 2 black balls, so the second contains 9 white and 5 black balls.
If pulled two balls at random, what is the probability of being both black?

The Properties of Probabilities

1. $P(B|B) = 1$ (*give example*)
2. $P(A \cup C|B) = P(A|B) + P(C|B)$; for $A \cap C = \phi$
3. If $A \subseteq B$, then $A \cap B = A$ so $P(A|B) = P(A)/P(B)$, for $A \subseteq B$ *(give example).*

Example

A die is rolled and the number shown is recorded.
If the number appearing is even,
1. What is the probability that it is six?

Let E denote the event “even number” and F denote the event “a six”.

That is, $F \subseteq E$,

$P(F|E) = P(F)/P(E) = 1/6|1/2 = 1/3$.

2. What is the probability that it is a 3?
3. What is the probability that it is a 5?
4. What is the probability that it is a 2?

The Addition Law

1- If the events are mutually exclusive

$P(A+B) = P(A)+P(B)$

Example:- Box contains on 4 black, 5 white, and 3 red ball. If we pulled one ball what is the prob, that black or white?

$P(W+B)=P(W)+P(B)$

$=4/12+5/12$

$=9/12.$

what is the prob, that black or red ?

2- If the events are independent or not mutually exclusive

$P(A+B) = P(A)+P(B)- P(A\cap B)$

Example:- If the probability of man A infecting the goal is ¼, and the probability of man B infecting the same goal is 2/5 .what is the probability of infecting the goal if A and B are toward to goal?

$P(A+B) = P(A)+P(B)- P(A\cap B)$

By $P(AB) = P(A)P(B)$ (A and B are independents)

$=1/4.2/5=1/10$

$P(A+B) = 1/4+2/5-1/10$

$=11/20.$

The multiplication law

1- If the events A and B are independent
P(AB) =P(A) .P(B)

Example: - two boxes, the first contains of 4 white and 2 black balls, while the second contains 3 white and 5 black balls. if a ball is pulled from both boxes .what is the probability are black?

p (B_1B_2) =P (B_1) P (B_2)
2/6.5/8=10/48.

2- If the events A and B are not independent
P(AB) =P(A) .P(B|A)

For general
P $(A_1 ,A_2 \ldots., A_n)$= P(A_1) ,$P(A_2|A_1)P(A_3|A_1A_2)$ … …P$(A_n|A_1 ,A_2 \ldots., A_{n-1})$. (Give example).

Example:- Box contains 5 red and 10 black balls if pulled two balls what is the probability are black?
P (B_1)=10/15
p $(B_2|B_1)$ =9/14
P (B_1B_2)= P (B_1)p $(B_2|B_1)$=10/15.9/14=???

If draw ten balls what the prob, are black?
If draw ten balls what the prob, 5 are black 5 are red?

Example: - A box contains 8 red , three white balls and 9 blue balls, if 3 balls are randomly pulled, calculate the probability.

1- The three are red.
2- The three are white
3- 2 red and one white

4- At least one is white.

5- One from each color.

6- The balls are pulled in sequence form (red , white and blue)

Solution

R_1 the first is red, R_2 second is red, R_3 third is red.

1- The events are not independent.

P($R_1R_2R_3$)= P(R_1). P($R_2|R_1$).P($R_3|R_1R_2$)= 8/20.7/19.6/18.14/285

Or P(3R)=$\frac{\text{Number choice 3 red balls from 8 balls}}{Number\ choice\ 3\ balls\ from\ 20\ balls} = \frac{\binom{8}{3}}{\binom{20}{3}} = 14/28$

2- P(3W) $= \frac{\binom{3}{3}}{\binom{20}{3}} = 1/1140$

3- P(2R1w) $= \frac{\binom{8}{2}\binom{3}{1}}{\binom{20}{3}} = 7/95$

4- P is not white P(W) $= \frac{\binom{17}{3}}{\binom{20}{3}} = 34/57$ OR P (at least one white).=1-P($\overline{w}$)

$= \binom{23}{57}$

Or P at least one white = P(1W2$\overline{w}$)+P(2W1$\overline{w}$)+ P(3W)= $\frac{\binom{3}{1}\binom{17}{2}}{\binom{20}{3}} + \frac{\binom{3}{2}\binom{17}{1}}{\binom{20}{3}} + \frac{\binom{3}{3}}{\binom{20}{3}}$

5- P(1R1W1B)= $\frac{\binom{8}{1}\binom{3}{1}\binom{9}{1}}{\binom{20}{3}} = 18/95$

6- P($R_1w_2\ B_3$)= P(R_1)P($w_2|R_1$) P($B_3|R_1w_2$)= $\left(\frac{8}{20}\right)\left(\frac{3}{19}\right)\left(\frac{9}{18}\right) = \frac{3}{95}$.

Examples: -In the series circuit we assume that R_1, R_2can be ------ R_1———R_2——— in two possible states 1 or 0 denoting operative or defective then S= {(1, 1), (0, 1), (0, 0)}

If E1 is the event "the entire circuit is operative" then

E1= {(1, 1)}

E2 is the event "at least one ofR_1, R_2" is operative then

E2={ (1,1)(1,0)(0,1)} we notice that E1 and E2 are not mutually exclusive .

Example

The box contains 7 white balls and 3 black balls. If pulled from it 3 balls without return .what is prob, the balls white?

Sol:- A_1, A_2, A_3

$$P(A_1, A_2, A_3) = P(A_1)P(A_2|A_1)P(A_3|A_2A_1)$$

= (7/10) (6/9) (5/8) = 210/720.

Example:

a bag contains 7 red and 3 green marbles, if pulled two balls are without return

Lets denote the events R_1 *the first is red* R_2 *the second is red* G_1 *the first is green* G_2 *the second is green*

First marble second marble

p(R_1)=7/10

P($R_2|R_1$)=6/9 P(R_1R_2)=7/10*6/9 =42/90

p($G_2|R_1$)=3/9 p(R_1G_2)=7/10*3/9=21/90

p(G_1)=3/10

p($R_2|G_1$)=7/9 p(G_1R_2)=

p($G_2|G_1$) =2/9 p(G_1G_2)=

P($R_2|R_1$)= P(R_2R_1)/ P(R_1)

=42/90|7/10=2/3

Since , P(R_2R_1)= $\frac{\binom{7}{2}}{\binom{10}{2}} = \frac{42}{90} = \frac{7}{15}$

P(R_2R_1)= P(R_1R_2)????

a bag contains 7 red and 3 green marbles, if pulled two balls are without return

Sol:-

P($R_2|R_1$)= $\frac{\mathrm{P}(R_2R_1)}{p(R_1)}$

Number of way to test ball is red=$\binom{6}{2}$

Number of way to test two ball is red=$\binom{10}{2}$

$$\mathrm{P}(R_2R_1) = \frac{\binom{6}{2}}{\binom{10}{2}} = \frac{1}{3}$$

P(R_1)=6/10 , P($R_2\ |R_1$) $= \frac{1/3}{6/10} = 5/9.$

Example: The box contains 3 red balls, 2 black balls, and one green ball. pulled two balls. what is the probability the balls are black?

Sol:- two stage experiment
The events in the first stage
P(R) =3/6
P(B) =2/6
P(G)=1/6

The events in the second stage

P(R_1) ⟹ P(R_2 |R_1) =2/5

P(B_1) ⟹ P(B_2 |R_1)